Solving a Production and Inventory Model with a Minimum Lot Size Constrain

Knut Richter

Irena Okhrin

European University Viadrina Frankfurt (Oder)

Department of Business Administration and Economics

Discussion Paper No. 261

September 2007

ISSN 1860 0921

Solving a Production and Inventory Model with a Minimum Lot Size Constrain

Knut Richter and Irena Okhrin

Abstract: The paper deals with the analysis of a special dynamic production and inventory model. In this model logical restrictions to fulfill an accepted constant minimal level of the production lot size are incorporated, instead of keeping setup cost in the objective function, as it is common in many other models. Detailed optimality conditions are derived, which make possible the application of a simple dynamic programming recursion procedure.

Keywords: dynamic production-inventory model, minimum lot size, dynamic programming

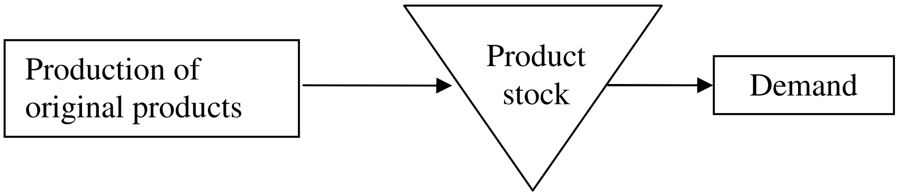
1. Introduction

In Richter and Gobsch (2005) a class of dynamic models of closed-loop logistics has been proposed in which the demand is satisfied by originally produced products as well as by re-manufactured used products. Apart from many other approaches (Minner and Lindner 2003, Richter and Sombrutzki 2000, Richter and Weber 2001, Golany, Yang, and Yu 2001, Beltrán and Krass 2002, etc.) the lot size is not determined by finding the minimum of the setup cost and holding cost but by minimizing the inventory cost subject to certain lot size restrictions. The special case of constant minimal lot size values and restricted return rates of an efficient dynamic programming algorithm has been presented which employs efficiently solvable combinatorial sub-problems. Since the main attention was paid to the problems of the closed-loop logistics, the algorithm was described rather shortly. In fact, the model described in that paper demands a two-dimensional dynamic programming procedure. Special assumptions on the backflow of used products, however, allow the reduction to a one-dimensional procedure. If the backflow is cut the dynamic problem with lot size restrictions to be studied here appears.

2. The basic problem with lot size restrictions

The process of producing a product for several periods of a planning horizon is considered. The items are produced during the periods according to the demand which occurs at the end of the periods. By accumulation of produced items stocks will be created and inventory cost appears (see Fig. 1).

Fig.1. *The basic problem*



The following symbols will be used below. The sets of integers, natural numbers and real numbers are denoted by I, $N = \{1,2,...\}$ and R, correspondingly. Furthermore, the following characters denote:

$T \in N$ – planning horizon consisting of T periods,

$D_t \in I$ – demand for the product in the t^{th} period, where $D_{i-1,t} = \sum_{j=i}^{t} D_j$, $D_{j,j} = 0$,

$H \in R$ – the per unit inventory cost for the product,

$m \in N$ – the minimal bound (level) of the positive lot size for the production process.

The variables are denoted by

I_t – inventory of final products at the end of the t^{th} period, $I_{i-1,t} = \sum_{j=i}^{t} I_j$ and

z_t – production rate of the t^{th} period $z_{i-1,t} = \sum_{j=i}^{t} z_j$.

Now the following model will be presented:

The inventory at the beginning and at the end of the planning horizon is set equal zero, e. g.

$$I_0 = I_T = 0 . \qquad (1)$$

The inventory of a new period equals the previous inventory plus the difference of production and demand:

$$I_t = I_{t-1} + z_t - D_t, I_t \geq 0, \ t = 1,2,...,T. \qquad (2)$$

Due to the given minimal lot size bound the real lot sizes have to be either equal zero or not to be smaller than m . This condition is expressed by

$$z_t \in \{0\} \cup [m,+\infty[, \quad t = 1,2,...,T . \qquad (3)$$

Lot size restrictions of the similar structure were also considered in Beer, Käschel, and Richter (1979) as well as in Richter, Bachmann and Dempe (1988). Recent papers that consider minimum lot size constrains are, among others, Kallrath (1999), Suerie (2005) and Souza, Zhao, Chen, and Ball (2004).

The goal of minimizing the inventory cost is modeled by the objective function:

$$H \cdot \sum_{t=1}^{T} I_t \to min \quad . \tag{4}$$

Note that the condition (3) forbids the constellation $0 < z_t < m$, i.e. no lot sizes below the minimal level are allowed. In the objective function (4) the end of period inventories are multiplied by the cost factors and the whole expression is to be minimized. Since that cost factor is not significant for the study, it will be omitted, and the minimization of the total inventory will remain as the main goal.

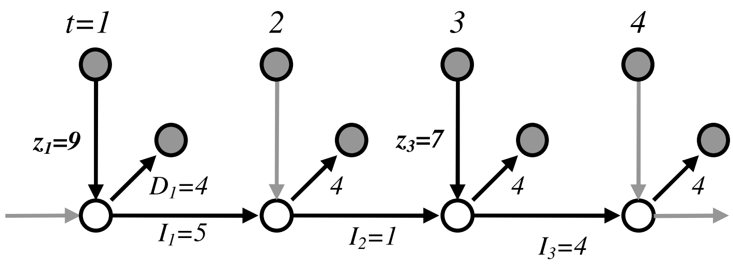
The entire model (1) – (4) has then the following compact form (5).

$$\begin{aligned} &I_0 = I_T = 0, \\ &z_t \in \{0\} \cup [m, +\infty[, \\ &I_t = I_{t-1} + z_t - D_t,\ I_t \geq 0,\ t = 1,2,...,T, \\ &\sum_{t=1}^{T} I_t \to \min \end{aligned} \tag{5}$$

Example 1: Given the demand $D = (4,4,4,4)$, $m = 7$. The solution No. 1 with two production periods shown in Fig. 2 is obviously feasible. The total inventory of this solution is $5+1+4=10$. This solution can be also presented graphically as in Fig. 3. The demand is shown by bold lines at the end of the periods, the productions quantities are represented by the marked areas and the inventories by grey areas.

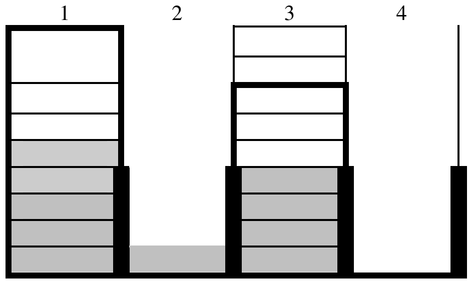
Better feasible solutions with the cost of $10-2\varepsilon$ units are given by the solutions No. 2(ε): $z_1 = 9-\varepsilon, z_2 = 7+\varepsilon, I_1 = 5-\varepsilon, I_2 = 1-\varepsilon, I_3 = 4, z_2 = z_4 = I_4 = 0$ for $0 \leq \varepsilon \leq 1$.

Fig. 2. *The material flow for the solution No. 1 of example 1*



The model (5) is obviously a linear program on a union of convex polyhedral sets. Hence an optimal solution can be found among the basis (extreme) solutions. The approach presented now will concentrate on the properties of basis solutions.

Fig. 3. *The solution No. 1 of example 1*



3. The Dynamic Programming Approach

3.1. Sub-problems with restricted lot sizes

The problem (5) will be denoted by $SP(0,T)$ and any sub-problem for selected periods $j = i, i+1, ..., t$ with $I_{i-1} = I_t = 0, I_j \geq 0, j = i, ..., t-1$ by $SP(i-1,t)$. It will be also said that a sub-problem is generated by the pair (i,t). The corresponding minimum cost for an optimal basis solution of a solvable sub-problem will be denoted by $C_{i-1,t}$. The sub-problem $SP(i-1,t)$ is called *solvable (unsolvable)* if $D_{i-1,t} \geq m \quad (D_{i-1,t} < m)$ (6)

holds. A sub-problem $SP(i-1,t)$ is called *minimal,* if there is no period $i < k < t$ such that

$$C_{i-1,t} = C_{i-1,k} + C_{k,t}. \quad (7)$$

It follows from the definition of the minimal sub-problem that the inventory values of an optimal solution are positive except for the last period.

An optimal basis solution of the problem (5) consists on a composition of optimal basis solutions for appropriate minimal sub-problems

$$SP(0,t_1), SP(t_1,t_2), ..., SP(t_{L-1},t_L),\ 1 \leq L \leq T. \quad (8)$$

The feasible basis solutions with positive inventories for the first example are given by $z^1_{0,4} = (16,0,0,0)$, $I^1_{0,4} = (12,8,4,0)$ and $z^2_{0,4} = (7,9,0,0)$, $I^2_{0,4} = (3,8,4,0)$ with the total inventory values *24* and *15*, correspondingly. This example regarded as a sub-problem is not minimal, since the minimum is reached by $C_{0,2} + C_{2,4} = 8$.

Transformations: In the argumentation used below three types of transformation are applied all of which produce a new feasible solution with reduced total inventory.

$T1(j', j'')$: In this case there are given two production periods $j' < j''$

with $z_{j'} > z_{j''} \geq m,\ I_j > 0, j = j',..., j'' - 1$. Then the transformation

$$z_{j'} := z_{j'} - \varepsilon, z_{j''} := z_{j''} + \varepsilon, I_j := I_j - \varepsilon, j = j',..., j'' - 1 \text{ for } \varepsilon = min\{z_{j'} - m, I_{j''-1}\} \quad (9)$$

will be applied.

$T2(j)$: There is a production period $j > i$ with $I_{j-1} \geq D_j$. Then the transformation

$$z_{j+1} := z_{j+1} + z_j,\ I_j := I_j - z_j, z_j = 0 \text{ will be applied.} \quad (10)$$

$T3(j)$: There is a production period $j < t$ with $z_j \geq 2m$ or $z_j = m$ and $I_j \geq m$. Then the transformation $z_j := z_j - m,\ I_j := I_j - m, z_{j+1} := z_{j+1} + m$ will be applied. (11)

The transformations are illustrated by the examples 2 in Tab. 1.

Tab. 1. Examples 2 to illustrate the transformations *T1 – T3*

j	1	2	3	4	5	6	Total inventory
D_j	4	4	4	4	4	4	
z_j	7	**10**	0	0	**7**	0	T1(2, 5)
I_j	3	9	5	1	4	0	22
z_j	7	**9**	0	0	**8**	0	↓
I_j	3	8	4	0	4	0	19
z_j	7	10	0	**7**	0	0	T2(4)
I_j	3	9	5	8	4	0	29
z_j	7	10	0	0	**7**	0	↓
I_j	3	9	5	1	4	0	22
z_j	7	**17**	0	0	0	0	T3(2)
I_j	3	16	12	8	4	0	43
z_j	7	**10**	**7**	0	0	0	↓
I_j	3	9	12	8	4	0	36

Lemma 1: Let an optimal basis solution of a minimal sub-problem $SP(i-1,t)$ be given. (i) Then there is no more than one period j' such that $z_{j'} > m$, i. e. $z_j \in \{0, m\},\ j \neq j'$. (12)

(ii) Such a period is the last production period for an optimal solution.

Proof: (i) Let an optimal basis solution be given with $j' < j''$ and

$z_{j'}, z_{j''} > m,\ I_j > 0, j = j',..., j'' - 1$. Then the transformation $T1(j', j'')$ can be applied which shows that the initial solution is not optimal or the sub-problem is not minimal.

(ii) In the case when $z_{j'} > m$ and there is a production period j'' after j', the application of $T1(j', j'')$ would reveal that the solution is not optimal.

If an optimal basis solution contains a period j' with $z_{j'} > m$, or if j' is the last production period with $z_{j'} = m$, the periods $j < j'$ for which $z_j = 0 \vee m$ hold will be called *extreme pro-*

duction periods. The periods $j > j'$, for which the values are equal zero i. e., $z_j = 0$, are called *zero production periods*.

3.1.1. Real sub-problems

A sub-problem will be called *real* if from the existence of a period $j^* = min\{j \geq i : D_{i-1,j} = l \cdot m, l \leq j - i + 1, l \in N\}$ it follows that $D_{j^*,t} < m$. (13)

If there is no such period then the problem is always real.

That means, provided a period $j*$ exists, then for a real sub-problem $D_{i-1,t} < (l+1) \cdot m$ holds:

Lemma 2: If a sub-problem $SP(i-1,t)$ is minimal then it is real.

Proof: Let an optimal basis solution of a minimal sub-problem be given which is not real, i.e. there is a period $j*$ and $D_{j*,t} \geq m$ holds. Let two cases be studied.

(i) If $z_j = 0 \vee m, j = i, i+1, ..., j*$ then either $I_{j*} = 0$ holds or $I_{j*} \geq m$. In the first case the sub-problem is not minimal, i.e. $C_{i-1,t} = C_{i-1,j*} + C_{j*,t}$. In the second case, due to $D_{j*,t} \geq m$, the transformation $T3(j)$ can be applied to the last production period $j' \leq j^*$ and hence the initial solution is not optimal.

(ii) There is a production period $j^+ \leq j*$ with $z_{j^+} > m$. Then according to Lemma 1 this is the last production period. Hence $z_j = 0 \vee m$ holds for $j < j^+$. Then due to the assumption of the proof the relation $I_{j^+} = D_{j^+,t} \geq m$ holds. Now two cases can be distinguished:

(α) If $z_{j^+} > 2m$ then the transformation $T3(j)$ can be applied for $j = j^+$.

(β) Let $z_{j^+} < 2m$. Then due to the definition of a real sub-problem (13) there is an integer $l > 0$ such that $l \cdot m = D_{i-1,j*}$. It will be proved now that the number of production periods with $z_j = m$ before j^+ is not less than the number l. If this is not true then $z_{i,j^+-1} + z_{j^+} < (l-1)m + 2m = D_{i-1,j*} + m \leq D_{i-1,t}$ holds, i.e. the whole demand will not be satisfied. Then $z_{i,j^+-1} \geq l \cdot m = D_{i-1,j^*}$ and $I_{j^+-1} = z_{i-1,j^+-1} - D_{i-1,j^+-1} \geq D_{j^+-1,j^*} \geq D_{j^+}$ hold.

That means that the transformation $T2(j)$ can be applied for $j = j^+$. □

Corollary: A real sub-problem is not necessarily minimal! Example 1 is real but not minimal.

3.1.2. Critical period

Below only minimal sub-problems will be analyzed. First, the following parameters and remarkable periods are introduced:

Let's introduce the integers $k_{i-1,j} = \left\lfloor \frac{D_{i-1,j}}{m} \right\rfloor$, $k_{i,i} = -1$. (14)

Provided, the relations $D_{i-1,l} < m \cdot (l - i + 1)$, $l = i, i+1, \ldots j$, $j < j^*$, (15)

hold, the number $k_{i-1,j} + 1$ is the minimal number of production periods of size m satisfying this demand. Since only one production figure can be larger than m, the number $k_{i-1,t}$ shows how many productions periods are needed to satisfy the whole demand of the sub-problem. Let the period $j^\# = min\{j \geq i : D_{i-1,j} > (j - i + 1) \cdot m\}$ (16)

be the first period where the relation (15) is strongly violated.

If the relations (15) hold, the *critical* period $j_{i-1,t} = min\{j \geq i : D_{i-1,j} > (k_{i-1,t} - 1) \cdot m\}$ (17)

is such a period the demand of which cannot be satisfied fully by $k_{i-1,t} - 1$ production periods of size m. Therefore it is the last production period with a production figure equal or larger than m. Alternatively, the definition $j_{i-1,t} = min\{j \geq i : k_{i-1,j} = k_{i-1,t} - 1\}$ (18)

can be used.

Another period which is important for the analysis is

$$j^\times = min\{j \geq i : k_{i-1,j} - k_{i-1,j-1} > 1\}. \quad (19)$$

If the periods j^*, $j^\#$ or $j^\times$ do not exist they will be set equal $t + 1$, correspondingly.

Lemma 3: Let a real sub-problem $SP(i-1,t)$ be given.

(i) Then $j_{i-1,t} \leq j^*$ and

(ii) if a period $j^\#$ exists then $D_{j^\#} > m$ and $j^\# \leq j^*$ holds and it coincides with $j^\times$. (20)

Proof: (i) Let $j_{i-1,t} > j*$. Then due to the definition (17) the relation $D_{i-1,j*} \le (k_{i-1,t} - 1) \cdot m$ and the inequality $D_{i-1,j*} + m \le k_{i-1,t} \cdot m \le D_{i-1,t}$ hold. It follows from the last inequality that $D_{j*,t} \ge m$, what contradicts the assumption.

(ii) Let now a period $j^{\#} > i$ exist. Then due to the definition (16) the relation $D_{j^{\#}} = D_{i-1,j^{\#}} - D_{i-1,j^{\#}-1} > (j^{\#} - i + 1)m - (j^{\#} - i)m = m$ holds and the demand $D_{j^{\#}}$ is larger than m. If there is a period $j*$, then the relation $t \le m(j^{*})$ holds. Then the case $j^{\#} > j*$ is not possible due to $D_{j^{\#}} > m$, i.e. $j^{\#} \le j*$ holds. It follows from this property and from the definition (16) that $D_{i-1,j^{\#}-1} < (j^{\#} - i) \cdot m$ and $\left\lfloor \frac{D_{i-1,j^{\#}-1}}{m} \right\rfloor \le j^{\#} - i - 1$. Based on this inequality the following estimation can be given:

$k_{i-1,j^{\#}} - k_{i-1,j^{\#}-1} = \left\lfloor \frac{D_{i-1,j^{\#}}}{m} \right\rfloor - \left\lfloor \frac{D_{i-1,j^{\#}-1}}{m} \right\rfloor \ge j^{\#} - i + 1 - (j^{\#} - i - 1) = 2$, by which the equality $j^{\#} = j^{\times}$ is proved. If $j^{\#} = i$ then the statement holds per definition. □

Corollary: Since due to Lemma 2 minimal sub-problems are real Lemma 3 holds for such problems, too.

Tab. 2: Example 3 of a real (minimal) sub-problem with $m = 7$, $k_{0,6} = 4$, $j^{\#} = j^{\times} = 4$, since $D_{0,3} = 18 < 4 \cdot 7 < 29 = D_{0,4}$ and $k_3 = 2 < 4 = k_4$

j	1	2	3	4	5	6	Total inventory
D_j	6	6	6	11	2	2	
$k_{0,j}$	0	1	2	4			
z_j	7	7	7	12	0	0	
I_j	1	2	3	4	2	0	12
z_j	7	11	0	15	0	0	
I_j	1	6	0	4	2	0	13

Lemma 4: Let a sub-problem $SP(i-1,t)$ be given with a period $j^{\#}$.

(i) If the sub-problem is real and $D_{j^{\#},t} < m$ then $j^{\#} = j_{i-1,t}$ and

(ii) if it is minimal then $D_{j^{\#},t} < m$ and $j^{\#} = j_{i-1,t}$.

Proof: If the sub-problem is real and a period j^{*} exists, then it follows from Lemma 3 that

$j^{\#} \le j^*$. Moreover, $D_{j^*,t} < m$ and $D_{i-1,t} \ge k_{i-1,t} \cdot m$ hold. Then $D_{i-1,j^*} = D_{i-1,t} - D_{j^*,t} > (k_{i-1,t} - 1) \cdot m$ and $j^* \ge j_{i-1,t}$.

(i) Let $D_{j^{\#},t} < m$. Then $k_{i-1,j^{\#}} = \left\lfloor \frac{D_{i-1,t} - D_{j^{\#},t}}{m} \right\rfloor \ge k_{i-1,t} - 1$ holds. (21)

It will be proved that $j^{\#} \ne j_{i-1,t}$ will lead to contradictions.

First, the case $j^* \ge j_{i-1,t} > j^{\#}$ will be analyzed. Then due to the definition (17) $D_{i-1,j^{\#}} < (k_{i-1,t} - 1)m$ and due to relation (21) the contradicting inequalities $k_{i-1,j^{\#}} < k_{i-1,t} - 1 \le k_{i-1,j^{\#}}$ hold.

Secondly, let the case $j_{i-1,t} < j^{\#} \le j^*$ be considered. Then due to the definition (16) the relation $D_{i-1,j_{i-1,t}} < (j_{i-1,t} - i + 1)m \le (j^{\#} - i)m$ holds. Due to this inequality and the definition (18) the relation $k_{i-1,j_{i-1,t}} = k_{i-1,t} - 1 < j^{\#} - i$ occurs. Due the definition (16) also $j^{\#} - i + 1 \le k_{i-1,j^{\#}} \le k_{i-1,t}$ holds that leads to the contradiction $k_{i-1,t} < j^{\#} - i + 1 \le k_{i-1,t}$.

Hence the first statement is true, i.e. $j^{\#} = j_{i-1,t}$.

(ii) Because of Lemma 2 this case is fulfilled for $D_{j^{\#},t} < m$ automatically. For $D_{j^{\#},t} \ge m$ it will be proved that the sub-problem is not minimal. Let an optimal basis solution be given. Since $j^{\#} - i + 1$ production runs of size m do not satisfy the demand $D_{i-1,j^{\#}}$, the production rate at the period $j^{\#}$ will fulfill $z_{j^{\#}} > m$. Then due to Lemma 1 this is the last production period at all and $z_{j^{\#}} = D_{i-1,j^{\#}} + D_{j^{\#},t} - z_{i-1,j^{\#}-1} > (j^{\#} - i + 1)m + m - (j^{\#} - i)m = 2m$. Furthermore, the estimate $I_{j^{\#}} = D_{j^{\#},t} \ge m$ holds and, the transformation $T3(j)$ can be applied. Hence the case (ii) is not valid for minimal sub-problems. □

Lemma 5: Let a sub-problem $SP(i-1,t)$ be given with a period $j^{\times}$.

(i) If the sub-problem is real and $D_{j^{\times}t} < m$ then $D_{j^{\times}} > m$, $j^{\times} \le j^*$ and $j^{\times} = j_{i-1,t}$ and,

(ii) if it is minimal then $D_{j^{\times}t} < m, D_{j^{\times}} > m$, $j^{\times} \le j^*$ and $j^{\times} = j_{i-1,t}$. (22)

Proof: If a period $j^{\#}$ exists, Lemmas 3 and 4 can be applied for both cases.

(i) Let $D_{j^{\times},t} < m$. Then, as in the proof of Lemma 4 (see formula (21)) the relation

$$k_{i-1,j^{\times}} = \left\lfloor \frac{D_{i-1,t} - D_{j^{\times},t}}{m} \right\rfloor \geq k_{i-1,t} - 1 \text{ holds.} \tag{23}$$

Further, it follows from $k_{i-1,j^{\times}} - k_{i-1,j^{\times}-1} \geq 2$ that $D_{j^{\times}} > m$. Hence as in the proof of Lemma 3 the inequality $j^{\times} > j^*$ is not possible.

First, let $j_{i-1,t} < j^{\times} \leq j^*$. Then $k_{i-1,j_{i-1,t}} < k_{i-1,j^{\times}} - 1$ and $D_{i-1,j_{i-1,t}} \leq (k_{i-1,t} - 1)m$ hold. That is, however, a contradiction to the definition (17).

Secondly, let $j^{\times} < j_{i-1,t} \leq j^*$. Then $D_{i-1,j_{i-1,t}} > (k_{i-1,t} - 1)m$, $D_{i-1,j^{\times}} < (k_{i-1,t} - 1)m$ and $k_{i-1,j^{\times}} < k_{i-1,t} - 1$. That is a contradiction to formula (23). Hence the first statement concerning $j^{\times} = j_{i-1,t} \leq j^*$ is true.

(ii) For the case of $D_{j^{\times},t} \geq m$, it will be proved that in this case the sub-problem is not minimal. Because of the assumption the sub-problem is real and the situation $j^{\times} > j^*$ is not possible due to $D_{j^{\times}} > m$. The equality $j^{\times} = j^*$ is not possible either, because of the assumption on (ii). Hence $j^{\times} < j^*$. Note, that in this case $k_{i-1,j^{\times}} < k_{i-1,t}$ holds. Now let some optimal basis solution be given.

First, let $z_{j^{\times}-1,t} = 0$. Then, since $D_{j^{\times}} > m$ and $D_{j^{\times},t} \geq m$ there is some closest period $j' < j^{\#}$ with $z_{j'} \geq m$ and $I_{j'} \geq D_{j^{\times}-1,t} > 2m$. If $z_{j'} \geq 2m$ then the transformation $T3(j)$ can be applied. Let $z_{j'} < 2m$. Then $2m < I_{j'} = I_{j'-1} + z_{j'} - D_{j'} < I_{j'-1} + 2m - D_{j'}$ and, $D_{j'} < I_{j'-1}$ and, the transformation $T2(j)$ can be applied.

Secondly, if $z_{j^{\times}} = 0,\ z_{j^{\times},t} > 0$ then there exists a closest period $j' < j^{\times}$ with $z_{j'} = m$ and $I_{j'} > m$. Then again the transformation $T3(j)$ can be applied.

Thirdly, if $z_{j^{\times}} \geq m,\ z_{j^{\times},t} = 0$ then either $z_{j^{\times}} \geq 2m,\ I_{j^{\times}} \geq m$ or $z_{j^{\times}} < 2m,\ I_{j^{\times}} \geq m$. In the first case the transformation $T3(j)$ can immediately be applied to the period $j^{\times}$. For the second case, it can be noticed that the inequalities $k_{i-1,j^{\times}-1} + 2 \leq k_{i-1,j^{\times}} \leq k_{i-1,t} - 1$,

$k_{i-1,j^\times-1} \le k_{i-1,t} - 3$ and $D_{i-1,j^\times-1} \le (k_{i-1,t} - 2)\cdot m$ hold. Furthermore the relations $z_{i-1,j^\times-1} + z_{j^\times} \ge k_{i-1,t}\cdot m$ and $-z_{j^\times} > -2m$ hold, i.e. $z_{i-1,j^\times-1} > (k_{i-1,t} - 2)\cdot m$ is fulfilled. Since $z_{i-1,j^\times-1}$ is a multiple of m, actually $z_{i-1,j^\times-1} \ge (k_{i-1,t} - 1)\cdot m$ holds. This means that $I_{j^\times-1} = z_{i-1,j^\times-1} - D_{i-1,j^\times-1} \ge m$ is true and the transformation $T3(j)$ can be applied to the last positive production period before $j^\times$. □

Lemma 5 is illustrated by the example 4 in Tab. 3. The solution with $D_{j^\times,t} \ge m$, $z_{j^\times} > 2m$ and $I_{j^\times} > m$ is transformed by $T3(j)$ and, a better solution appears as a composition of solutions for $SP(i-1, j^\times)$ and $SP(j^\times, t)$.

Tab. 3: Example 4 with $m = 7$, $k_{0,6} = 3$, $j^\times = 4$

j	1	2	3	4	5	6	Total inventory
D_j	2	2	2	9	4	4	
$k_{0,j}$	0	0	**0**	**2**			
z_j	7	0	0	**16**	0	0	
I_j	5	3	1	8	4	0	21
z_j	7	0	0	**8**	**8**	0	↓
I_j	5	3	1	0	4	0	13

Corollary: It can be seen that if such period $j^\times$ will be found then it will be always the last production period for a minimal sub-problem.

3.1.3. Critical solution

Now the following components of a basis solution for minimal sub-problems can be determined by means of the parameters (14) and (19):

$$I_j^* = (k_{i-1,j} + 1)\cdot m - D_{i-1,j},\ z_j^* = I_j^* + D_j - I_{j-1}^* \text{ for } j < j_{i-1,t} \qquad (24)$$

and $z^*_{j_{i-1,t}} = D_{j_{i-1,t}-1,t} - I^*_{j_{i-1,t}-1}$, $z_j^* = 0, j > j_{i-1,t}, I_j^* = D_{j,t}, j \ge j_{i-1,t}$. (25)

The basis solution (24) – (25) will be called *critical*. The total inventory provided by the critical solution is denoted by $C^*_{i-1,t}$.

Lemma 6: Let a sub-problem $SP(i-1,t)$ for given period i be regarded with $t = t_i$ and

$t_i = min\{m(j^{\#}), m(j^{*}), m(j^{\times})\}$, (26)

where $m(j) = max\{r : D_{j,r} < m\}$. Then the problem is real and the critical solution is feasible.

Proof: Because of the assumption (26) the sub-problem is real. Then Lemma 3 holds and $j_{i-1,t} \le j*$ is fulfilled and, due to the definition (13) the relation $k_{i-1,j} \cdot m < D_{i-1,j} < (k_{i-1,j} + 1) \cdot m$ holds for $j = i, i+1, ..., j_{i-1,t} - 1$. It follows from these inequalities that the inventory values (24) are valid, i.e. the strong inequalities $I_j^* = (k_{i-1,j} + 1) \cdot m - D_{i-1,j} > 0$, $j < j_{i-1,t}$ hold. In case of the existence of a period $j^{\#}$ the latter is equal $j_{i-1,t}$ and $j^{\#} \le j*$. That means that $D_{i-1,j} < (j - i + 1) \cdot m$ and $k_{i-1,j} + 1 \le j - i + 1$, $j < j^{\#}$, i.e. the number of production periods is not greater than the number of periods.

According to (24) the equality $z_j^* = I_j^* + D_j - I_{j-1}^* = (k_{i-1,j} - k_{i-1,j-1}) \cdot m$ (27)

holds. If there is a period $j^{\times}$ then due to Lemma 5 it is equal $j_{i-1,t}$ and therefore $k_{i-1,j} - k_{i-1,j-1} \le 1$, i.e. the production figures are equal zero or m. Furthermore, it follows from the definitions (24) – (25) and the inequality $D_{i-1,j_{i-1,t}-1} < (k_{i-1,t} - 1) \cdot m < D_{i-1,j_{i-1,t}}$ that $k_{i-1,j_{i-1,t}-1} + 1 \le k_{i-1,t} - 1$ and that $z_{j_{i-1,t}}^* = D_{j_{i-1,t}-1,t} - I_{j_{i-1,t}-1}^* =$

$= D_{j_{i-1,t}-1,t} + D_{i-1,j_{i-1,t}-1} - (k_{i-1,j_{i-1,t}-1} + 1) \cdot m \ge D_{i-1,t} - k_{i-1,t} \cdot m + m \ge m$, i.e. the production figure at the critical period is feasible. □

Now it will be shown that the critical solution is also optimal.

Theorem 1: Let a minimal sub-problem be given. Then the critical solution is optimal.

Proof: (i) Due to Lemma 1 an optimal solution has a last production period whose production figure can be larger than m. Therefore there is a period $J_{i-1,t}$ as the last production period and the periods before $J_{i-1,t}$ are extreme production periods and the periods after $J_{i-1,t}$ are zero production periods per definition. The inventories values for the periods after $J_{i-1,t}$ are obviously given by $I_j = D_{j,t}, j = J_{i-1,t}, ..., t$. Now some more properties of optimal solutions will be proved.

(ii) It will be shown that for an optimal solution the relation $z_j > 0$ implies $I_{j-1} < min\{m; D_j\}$, or equivalently $I_{j-1} \geq min\{m; D_j\}$ implies $z_j = 0$. Let j be the first period with $z_j > 0$ and $I_{j-1} \geq min\{m; D_j\}$.

(iia) If $m \leq D_j$, and $I_{j-1} \geq m$ then $j > i$. Then there is a nearest production period $j' < j$ for which due (i) $z_{j'} = m$ holds. Since the sub-problem is also real, due to Lemma 2, the inventory value cannot be equal m and, it is therefore strongly larger than m, i.e. $I_{j-1} > m$ holds. Then the relation $I_{j'} = D_{j',j-1} + I_{j-1} > D_{j',j-1} + m$ holds and the transformation $T3(\ j\ )$ can be applied to the period j'.

(iib) If $m > D_j, z_j > 0$ and $I_{j-1} \geq D_j$, then the transformation $T2(\ j\ )$ can be applied.

(iii) Now it will be shown that $I_{J_{i-1,t}} < max\{2m; 3m - D_{J_{i-1,t}}\}$. Let $j = J_{i-1,t}$ and, on the opposite, $I_j \geq \Delta = max\{2m; 3m - D_j\}$ and $z_j \geq m$.

(iiia) If $D_j > m$, i.e. $\Delta = 2m$ and $2m \leq I_j = I_{j-1} + z_j - D_j$, then due to (ii) $2m < m + z_j - D_j$ holds and $2m < D_j + m < z_j$. Then the transformation $T3(\ j\ )$ can be applied to the period $j = J_{i-1,t}$.

(iiib) If $D_j \leq m$, i.e. $\Delta = 3m - D_j$, then due to (ii) $3m - D_j \leq I_j < m + z_j - D_j$ holds and $2m < z_j$. That means that the same transformation can be applied!

(iv) Finally, it will be shown that $J_{i-1,t} = j_{i-1,t}$ and that the inventory values for the extreme production periods are given by $I_j = I_j^*$, i.e. the optimal solution coincides with the critical solution. The critical solution is feasible due to Lemma 6.

(iva) Let now $I_j < I_j^*$ be fulfilled for some period $j \leq J_{i-1,t}$. Then, however, there is at least one production period less, i.e., $I_j \leq k_{i-1,j} \cdot m - D_{i-1,j} < 0$ holds and the solution is not feasible.

(ivb) If $I_j > I_j^*$ holds for a first production period j, there is at least one production period more and $I_j \geq (k_{i-1,j} + 2) \cdot m - D_{i-1,j}$ and $I_{j-1} = (k_{i-1,j-1} + 1) \cdot m - D_{i-1,j-1}$. Then $I_j \geq m$ and $D_{j,t} \geq m$ holds. Due to Lemma 5(ii) then $j < j^{\times} \leq j^*$ holds.

Then one of the two options is fulfilled: a) $k_{i-1,j} = k_{i-1,j-1}$ or, b) $k_{i-1,j} = k_{i-1,j-1} + 1$.

In the case a) per definition (14) $D_{i-1,j} < k_{i-1,j-1} \cdot m$, $D_j < m$ and,

$I_j \geq (k_{i-1,j-1} + 2) \cdot m - D_{i-1,j} > 2m$ holds.

If a1) $j = J_{i-1,t}$ then this is a contradiction to the statement (iii).

If a2) $j < J_{i-1,t}$ then $z_j = m$ and $I_{j-1} = I_j - z_j + D_j > D_j$ which is a contradiction to (ii).

In the case b) $I_j = I_{j-1} + z_j - D_j \geq (k_{i-1,j-1} + 3) \cdot m - D_{i-1,j-1} - D_j = I_{j-1} + 2m - D_j$ holds, or $z_j \geq 2m$. Then, however $j = J_{i-1,t}$.

Hence $I_j = I_j^*$ holds for $j \leq J_{i-1,t}$. Then $J_{i-1,t} = j_{i-1,t}$ since the critical solution cannot contain another production period. □

Remark: It follows from the previous statements that for a given period i the maximal length of a minimal sub-problem can be estimated by (26). Therefore the feasible pairs (i,t) which generate minimal sub-problems belong to the set

$$X = \{(i,t) : i < t \leq t_i, (m(t) < T \vee t = T), (i = 1 \vee i > m(0))\}. \tag{27}$$

Due to the Lemma 6 sub-problems defined on elements from set X are real and the critical solution is feasible.

3.2. Solution algorithm

The approach presented here utilizes the ideas of dynamic programming and is illustrated by the example 5 in Tab. 4.

Let F_t be the minimal cost for the first t periods. Furthermore, let $F_0 = 0$. An optimal solution for the problem (5) can be found by the recursion

$$F_t = \min_{(i,t) \in X} \{C^*_{i-1,t} + F_{i-1}\} = C^*_{i(t)-1,t} + F_{i(t)}. \tag{28}$$

Detailed algorithm:

$$\begin{aligned}
&F_0 := 0; \text{ For } t := 1 \text{ until } T \text{ do } F := +\infty; \\
&\text{For } i := 1 \text{ until } t \\
&\text{do if } (i,t) \in X \text{ and } F > F_{i-1} + C^*_{i\text{-}1,t} \\
&F := F_{i-1} + C^*_{i\text{-}1,t};\ i(t) := i \text{ end; end}
\end{aligned} \tag{29}$$

Lemma 7: The algorithm (29) generates an optimal solution as a series of optimal solutions of minimal sub-problems.

Proof: First, it will be proved that the pairs $(i(t),t)$ generate minimal sub-problems. If this is not true then there are two first periods t and i_1 such that $C^*_{i(t)-1,t} \geq C^*_{i(t)-1,i_1} + C^*_{i_1-1,t}$. (30)

Due to the definition of $i(t)$ the inequality $F_{i(t)} + C^*_{i(t)-1,t} < F_{i_1} + C^*_{i_1-1,t}$ holds. Furthermore the relation $F_{i_1} \leq F_{i(t)} + C^*_{i(t)-1,i_1}$ holds and

$F_{i(t)} + C^*_{i(t)-1,t} < F_{i(i)} + C^*_{i(i)-1,i_1} + C^*_{i_1-1,t}$. This is a contradiction to inequality (30).

Secondly, it will be proved that minimal total inventory for a problem over the first t periods is given by the value F_t. The sub-problem $SP(0,m(0)+1)$ is obviously minimal and it has the critical solution $z^*_1 = D_{0,m(0)+1}, z^*_j = 0, 1 < j \leq m(0)+1$.

Hence $F_{m(0)+1} = C^*_{0,m(0)+1} + F_0$ express the minimal total inventory for $t = m(0)+1$ periods. Let now t be any period and the statement true for any period smaller than t. If F_t is not equal the minimal total inventory then there is some period $k < t$ such that

$$F_k + C_{k-1,t} < F_{i(t)} + C^*_{i(t)-1,t}. \quad (31)$$

Then, however $C_{k-1,t} = C_{k-1,i_1} + C^*_{i_1-1,t}$ and $F_k + C_{k-1,i_1} = F_{i_1}$ holds. The latter relation leads to a contradiction to the assumption (31). □

4. Conclusions

A special dynamic production and inventory model has been studied in this paper. Apart from the mainstream of lot-size modeling here logical restrictions to keep the lot size on an accepted minimal level will contribute to set up an efficient production plan. The detailed analysis of the model allowed formulating a rather simple solution procedure. Like in the classical lot-sizing theory generalizations of the model will lead to NP-hard problems, but there is yet a hope to find efficient solutions for models with an upper bound of the lot size.

Tab. 4: Dynamic programming

m=7, t=	1	2	3	4	5	F_i+$C_{i-1,t}$	F_t
D_t	6	6	6	11	6	-	-
z_t	12	0					
I_t	6	0				0+6	6
z_t	7	11	0				
I_t	1	6	0			0+7	7
z_t	7	7	15	0			
I_t	1	2	11	0		0+14	
z_t			7	10			
I_t			1	0		6+1	7
z_t	7	7	7	14	0		
I_t	1	2	3	6	0	0+12	
z_t			7	16	0		
I_t			1	6	0	6+7	
z_t				17	0		
I_t				6	0	7+6	**13**

Literature

Beer, K., J. Käschel, and K. Richter 1979. Ein Produktionsplanungsmodell bei horizontaler und vertikaler Verflechtung, Mathematische Operationsforschung und Statistik, Series Optimization 10 (1), 85–92.

Beltrán J. L. and D. Krass 2002. Dynamic Lot Sizing with Returning Options, IIE Transactions 34, 437–448.

Golany B., J. Yang, G. Yu 2001. Economic Lot-Sizing with Remanufacturing Options, IIE Transactions 33, 995–1003.

Kallrath, J. 1999. The Concept of Contiguity in Models Based on Time-Indexed Formulations, in: Keil, F., W. Mackens, and J. Werther (eds.), Scientific Computing in Chemical Engineering II, Springer, Berlin, 330–337.

Minner S, G. Lindner (2003): Lot sizing in product recovery decisions, in: Dekker R., M. Fleischmann, K. Inderfurth, and L. van Wassenhove (eds.), Reverse Logistics. Quantitative Models for Closed-Loop Supply Chains, Springer, Berlin, 157–179.

Richter, K., P. Bachmann, and S. Dempe 1988. Diskrete Optimierungsmodelle. Effektive Algorithmen und Näherungslösungen, VEB Verlag Technik, Berlin.

Richter K., B. Gobsch 2005. Kreislauf-Logistik mit Losgrößenrestriktionen, Zeitschrift für Betriebswirtschaft, ZfB-Special Issue 4/2005, 57–79.

Richter K., M. Sombrutzki 2000. Remanufacturing Planning for the Reverse Wagner/Whitin Models, European Journal of Operational Research 121 (2), 304–315.

Richter K., J. Weber 2001. The Reverse Wagner/Whitin Model with Variable Manufacturing and Remanufacturing Cost, International Journal of Production Economics 71, 447–456.

Souza, G.C., Z. Zhao, M. Chen, and M.O. Ball 2004. Coordinating Sales and Raw Material Discounts in a Global Supply Chain, Production and Operations Management 13 (1), 34–45.

Suerie, C. 2005. Campaign Planning in Time-Indexed Model formulations, International Journal of Production Research 43 (1), 49–66.

Diskussionspapiere der Europa-Universität Viadrina Frankfurt (Oder) Fakultät Wirtschaftswissenschaften (ab 2000*)

146. **Jan Winiecki:** Successes of Trade Reorientation and Trade Expansion in Post - Communist Transition: an Enterprise - Level Approach. Januar 2000.
147. **Jan Winiecki:** Cost and Benefits of European Union´s Enlargement: a (largerly) Sanguine View. Januar 2000.
148. **Alexander Kritikos**: The Enforcement of Enviromental Policy under Incomplete Information. Januar 2000.
149. **Stefan Schipper und Wolfgang Schmid:** Trading on the Volatility of Stock Prices. Januar 2000.
150. **Friedel Bolle und Alexander Kritikos:** Solidarity. Januar 2000.
151. **Eberhard Stickel:** Entrepreneur or Manager: Who really runs the Firm?. Februar 2000.
152. **Wolfgang Schmid und Stefan Schipper:** Monitoring Financial Time Series. Februar 2000.
153. **Wolfgang Schmid und Sven Knoth:** Kontrollkarten für abhängige Zufallsvariablen. Februar 2000.
154. **Alexander Kritikos und Frank Wießner:** Ein zweiter Kreditmarkt für eine zweite Chance. Februar 2000.
155. **Alexander Kritikos:** A Discussion on the Viability of the Indenture Game, between G. Holt and F. Bolle and A. Kritikos. März 2000.
156. **Claudia Kurz**: Regional Risk Sharing and Redistribution by the Unemployment Insurance: The Case of Germany. April 2000.
157. **Friedel Bolle und Andreas Paul**: Preventing International Price Discrimination – Are Fines Welfare Enhancing?. Mai 2000.
158. **Dorothea Baun:** Operationalisierung der Determinanten von Impulskäufen – Ergebnisse einer empirischen Untersuchung. Mai 2001.
159. **Alexander Haupt:** Environmental Policy and Innovations in Open Economies. Juni 2000.
160. **Jochen Hundsdoerfer:** Lock-In-Effekte bei Gewinnen von Kapitalgesellschaften vor und nach der geplanten Steuerreform. Juni 2000.
161. **Alexander Kritikos und Friedel Bolle**: Distributional Concerns: Equity or Efficiency Oriented?. Juli 2000.
162. **Sandra Große und Lars-Olaf Kolm**: Anrechnung nach § 34c Abs. 1 oder Abzug nach § 34c Abs. 2 EStG – Modellierung einer Entscheidungshilfe. August 2000.
163. **Swantje Heischkel und Tomas Oeltze:** Grundzüge des russischen Umsatzsteuerrechts. August 2000.
164. **Friedel Bolle:** Do you really want to know it?. September 2000.
165. **Friedel Bolle und Alexander Kritikos**: Reciprocity, Altruism, Solidarity: A dynamic model. September 2000.

* Eine Übersicht über die zwischen 1993 bis 1999 erschienenen Diskussionspapiere kann beim Dekanat der Wirtschaftswissenschaftlichen Fakultät angefordert werden.

166. **Jan Winiecki**: An inquiry into the early drastic fall of output in post-communist transition: An unsolved puzzle. Oktober 2000.
167. **Jan Winiecki**: Post Foreign Trade Problems and Prospects: The Economics and Political Economy of Accession. Januar 2001.
168. **Helmut Seitz**: Demographischer Wandel und Infrastrukturaufbau in Berlin-Brandenburg bis 2010/2015: Herausforderungen für eine strategische Allianz der Länder Berlin und Brandenburg. März 2001.
169. **Wolfgang Schmid und Yarema Okhrin:** Tail Behaviour of a General Family of Control Charts. April 2001.
170. **Jan Winiecki:** Polish Generic Private Sector In Transition: Developments And Characteristics. Juni 2001.
171. **Nadejda Pachomova, Alfred Endres und Knut Richter:** Proceedings des ersten Seminars über Umweltmanagement und Umweltökonomie. Mai 2001.
172. **Maciej Rosolowski and Wolfgang Schmid:** Ewma charts for monitoring the mean and the autocovariances of stationary gaussian processes. Juli 2001.
173. **Sven Knoth und Wolfgang Schmid:** Control Charts for Time Series: A Review. Oktober 2001.
174. **Adam Gieralka:** Die Unternehmenssteuerreform 2001 und die fundamentale Bewertung von Kapitalgesellschaften. Oktober 2001.
175. **Friedel Bolle:** If you want me, I don't want you. December 2001.
176. **Friedel Bolle:** Signals for Reliability: A possibly harmful institution? December 2001.
177. **Tomas Oeltze und Swantje Heischkel:** Das neue Umsatzsteuergesetz in der Russischen Föderation. Dezember 2001.
178. **Andrea Gröppel-Klein and Dorothea Baun:** The more the better? – Arousing merchandising concepts and in-store buying behavior. Februar 2002.
179. **Yves Breitmoser:** Collusion and Competition in Laboratory Simultaneous Multiple-Round Auctions. Mai 2002.
180. **Alexander Kritikos and Friedel Bolle:** Utility versus Income Based Altruism – in Favor of Gary Becker. Mai 2002.
181. **Elzbieta Kuba and Friedel Bolle:** Supply Function Equilibria under Alternative Conditions with Data from the Polish Electricity Market. Mai 2002.
182. **Friedel Bolle:** Altruism, Beckerian Altruism, or Intended Reciprocity? Remarks on an Experiment by Selten and Ockenfels. Mai 2002.
183. **Yves Breitmoser:** Subgame-Perfect Equilibria of Small Simultaneous Multiple-Round Auctions. Juni 2002.
184. **Yves Breitmoser:** Moody Behavior in Theory, Laboratory, and Reality. Juni 2002.
 - Diskussionspapier wurde zurückgezogen und wird neu überarbeitet -
185. **Antje Baier und Friedel Bolle:** Zyklische Preisentwicklung im offenen Call-by-Call-Markt: Irreführung der Konsumenten?. Oktober 2002.
186. **Yves Breitmoser:** Long-term Equilibria of Repeated Competitive Games. Januar 2003.
187. **F. Bolle und J. Kaehler: "**The Conditional Efficiency of Signaling. An Experimental Investigation." Frankfurt (Oder). October 2002.
188. **Friedel Bolle**, „The Envelope Paradox, the Siegel Paradox, and the Impossibility of Random Walks in Equity and Financial Markets". February 2003.

189. Friedel Bolle and Jessica Kaehler, "Is there a Harmful Selection Bias when Experimenters Choose their Experiments?". February 2003.

190. Helmut Seitz: Die langfristige Entwicklung der Einnahmen der Kommunen im Land Brandenburg vor dem Hintergrund der Ergebnisse der Solidarpakt-Verhandlungen unter besonderer Berücksichtigung der Zuweisungen des Landes. März 2003.

191. Thomas Otte: Die Praxis der Arbeitsbewertung in polnischen Unternehmen. April 2003.

192. Tomas Oeltze/Swantje Heischkel: Die Struktur des russischen Körperschafts-steuergesetzes. April 2003.

193. Knut Richter/Barbara Gobsch (Hrsg.): Proceedings des 2. deutsch-russischen Workshops zum Thema: Aktuelle Fragen und Trends der Wirtschaftswissenschaften. Mai 2003.

193. под ред. К. Рихтера/Б. Гобш: Материалы 2-ого немецко-русского семинара «Актуальные вопросы и тенденции экономической науки». май 2003.

194. Thomas Otte: Die arbeitgeberseitige Finanzierung der MBA-Ausbildung als Investition in Humankapital. Juni 2003.

195. Lars-Olaf Kolm: Die Konvergenz der Rechnungslegungsstandards: Eine stille Revolution IAS, die realistischere Bilanzierung?. Juni 2003.

196. Sven Knoth: Accurate ARL computation for EWMA-S^2 control charts. June 2003.

197. Sven Knoth: EWMA schemes with non-homogeneous transition kernels. June 2003.

198. Alfred Kötzle u. a.: Standortvorteile in Ostbrandenburg/Westpolen für grenzüberschreitende Kooperation. Juli 2003.

199. Thomas Otte: Das französische Hochschulsystem als Sortiereinrichtung für Humankapital. August 2003.

200. M. Rosołowski and W. Schmid: EWMA charts for monotoring the mean and the autocovariances of stationary processes. August 2003.

201. Adrian Cloer: Die Grundzüge des polnischen Einkommenssteuerrechts 2003. September 2003.

202. Jonathan Tan and Daniel J. Zizzo: Groups, Cooperation and Conflict in Games, October 2003.

203. Sven Knoth: Computation of the ARL for CUSUM-S^2 schemes, November 2003.

204. Jonathan Tan: Religion and Social Preferences: An Experimental Study. Januar 2004.

205. Adrian Cloer: Eine fallorientierte Einführung in das polnische Ertragsteuerrecht (einschließlich DBA-Rechts). Februar 2004.

206. Adam Gieralka: Steuerliche Vorteilhaftigkeit der Zwischenschaltung einer vermögensverwaltenden Kapitalgesellschaft – eine Fallstudie zum steueroptimalen Bezug polnischer Einkünfte in Deutschland unter Berücksichtigung der Hinzurechnungsbesteuerung nach §§ 7 – 14 AStG -. Februar 2004.

207. Friedel Bolle und Yves Breitmoser: Dynamic Competition with Irreversible Moves: Tacit Collusion (Almost) Guaranteed. Februar 2004.

208. Andrea Gröppel-Klein and Claas Christian Germelmann: Is Specific Consumer Behaviour Influenced by Terminal Values or does Yellow Press Set the Tone?. Februar 2004.

209. Andrea Gröppel-Klein and Anja Domke: The Relevance of Living-Style Match for Customer Relationship Marketing of Housing Companies. Februar 2004.

210. Michael Grüning and Kathalin Stöckmann: Corporate Disclosure Policy of German DAX-30 Companies. März 2004.

211. Elena Klimova: Ergebnisse einer Unternehmensumfrage zum Thema: "Betriebliches Umweltmanagement und Wertsteigerung im Unternehmen: Gegensätze oder zwei Namen für eine Erfolgsstrategie?". März 2004.

212. Thomas Otte: Dynamische Aspekte von Differenzierungsstrategien, April 2004.

213. Olha Bodnar and Wolfgang Schmid: CUSUM Control Schemes for Multivariate Time Series. April 2004.

214. Jonathan Tan and Friedel Bolle: On the Relative Strenghts of Altruism and Fairness. Mai 2004.

215. Susanne Leist: Integration von Techniken verschiedener Methoden der Unternehmensmodellierung, Arbeitspapier des Lehrstuhls für Allgemeine Betriebswirtschaftslehre, insbesondere Wirtschaftsinformatik, Finanz- und Bankwirtschaft der Europa-Universität Viadrina. Juni 2004.

216. Susanne Leist: Methoden der Unternehmensmodellierung – Möglichkeiten und Grenzen ihrer Anwendung, Arbeitspapier des Lehrstuhls für Allgemeine Betriebswirtschaftslehre, insbesondere Wirtschaftsinformatik, Finanz- und Bankwirtschaft der Europa-Universität Viadrina. Juni 2004.

217. Susanne Leist und Krzysztof Woda: Analyse der Erfolgsfaktoren mobiler Zahlungssysteme, Arbeitspapier des Lehrstuhls für Allgemeine Betriebswirtschaftslehre, insbesondere Wirtschaftsinformatik, Finanz- und Bankwirtschaft der Europa-Universität Viadrina. Juli 2004.

218. Jonathan H. W. Tan and Friedel Bolle: Team Competition and the Public Goods Game. Juli 2004.

219. Jonathan H. W. Tan and Anders Poulsen: The Role of Information in Ultimatum Bargaining. Juli 2004.

220. Olha Bodnar and Wolfgang Schmid: Multivariate Control Charts based on a Projection Approach. Oktober 2004.

221. Irena Okhrin and Knut Richter: Inventory and Transportation Models in the Mobile Business Environment. Oktober 2004.

222. Michael Krohn: Die virtuelle Falle - Konfliktpotentiale der Informationsgesellschaft und ihre Überwindung durch Investitionen in Sozialkapital. November 2004.

223. Friedel Bolle, Jana Heimel and Claudia Vogel: Crowding Out and Imitation Behavior in the Solidarity Game. Dezember 2004.

224. Andrea Gröppel-Klein and Claas Christian Germelmann: The Impact of Terminal Values and Yellow Press on Consumer Behavior. Januar 2005.

225. Friedel Bolle and Antje Baier: Cyclical Price Fluctuations caused by Information Inertia - Evidence from the German Call-by-Call Telephone Market. Januar 2005.

226. Grigori Pichtchoulov and Knut Richter: Economic Effects of Mobile Technologies on Operations of Sales Agents. Januar 2005.

227. **Jens Jannasch:** Erfolgsfaktoren mobiler, integrierter Geschäftsprozesse. Januar 2005.
228. **Michael Grüning, Kathalin Stöckmann and Marek Maksymowicz:** A Comparison of Corporate Disclosure in Germany and Poland. Februar 2005.
229. **Friedel Bolle and Alexander Kritikos:** Altruistic Behavior Under Incomplete Information, Februar 2005.
230. **Alexander S. Kritikos:** The Impact of Compulsory Arbitration on Bargaining Behavior – An Experimental Study - . Februar 2005.
231. **Alexander S. Kritikos and Denitsa Vigenina:** Key Factors of Joint-Liability Loan Contracts an Empirical Analysis. Februar 2005.
232. **Alexander S. Kritikos and Friedel Bolle:** Utility-Based Altruism: Evidence in Favour Gary Becker. Februar 2005.
233. **Alexander S. Kritikos, Friedel Bolle and Jonathan H. W. Tan:** The Economics of Solidarity: A Conceptual Framework. Februar 2005.
234. **Thomas Otte:** Personalwirtschaftliche Rahmenbedingungen und Strategien bei der Marktbearbeitung in Transformationsländern am Beispiel Polens. Februar 2005.
235. **Thomas Otte:** Das französische Hochschulsystem als Sortiereinrichtung für Humankapital. Februar 2005.
236. **Jonathan Tan and Daniel J. Zizzo:** Which is the More Predictable Gender? Public Good Contributionand Personality. März 2005.
237. **Maciej Wojtaszek und André Winzer:** Praxisnahe Steuerbilanzpolitik unter Berücksichtigung des Zinseffektes (veröffentlicht im EWZ). Mai 2005.
238. **Sven Husmann:** On Estimating an Asset's Implicit Beta. Juni 2005.
239. **Adam Gieralka:** Neue Runde im Kampf um Steuerquellen. Eine fallbezogene Analyse der Steuerfolgen aus dem Einsatz einer polnischen Zwischengesellschaft für eine deutsche Kapitalgesellschaft unter expliziter Berücksichtigung aktueller Steuerreformvorschläge, insbesondere des geplanten Gesetzes zur Verbesserung der steuerlichen Standortbedingungen vom 4. Mai 2005 (Europäisches Wissenschaftszentrum am Collegium Polonicum). Juni 2005.
240. **Jonathan H. W. Tan and Claudia Vogel:** Religion and Trust: An Experimental Study. Juli 2005.
241. **Alexander S. Kritikos and Jonathan H.W. Tan:** Indenture as a Commitment Device in Self-Enforced Contracts. August 2005.
242. **Adam Gieralka:** Die Hinzurechnungsbesteuerung als ein Weg aus der Falle des Halbeinkünfteverfahrens?. August 2005.
243. **Michael Grüning:** Divers of Corporate Disclosure – An Empirical Investigation in a Central European Setting. Oktober 2005.
244. **Andrea Gröppel-Klein, Claas Christian Germelmann, Martin Glaum:** Polnische und Deutsche Studierende an der Wirtschaftswissenschaftlichen Fakultät der Europa-Universität Viadrina: Ein Längsschnittsvergleich 1998 - 2004. Oktober 2005.
245. **Tessa Haverland:** Anonymity matters - Zur Relevanz einer Anonymitätsbetrachtung in den Wirtschaftswissenschaften. Dezember 2005.
246. **Sven Husmann, Martin Schmidt, Thorsten Seidel:** The Discount Rate: A Note on IAS 36. Februar 2006.
247. **Yves Breitmoser:** A Theory of Coalition Bargaining. Februar 2006.

248. **Volodymyr Perederiy:** Insolvenzprognose anhand von ukrainischen handelsrechtlichen Abschlüssen: explorative Analyse. August 2006.

249. **Alexander Kritikos and Friedel Bolle:** Utility versus Income-Based Altruism. September 2006.

250. **Friedel Bolle:** A Price is a Signal – On Intrinsic Motivation and Crowding – out. September 2006.

251. **Alexander Kritikos, Christoph Kneiding and Claas Christian Germelmann:** Is there a Market for Microlending in Industrialized Countries?. September 2006.

252. **Marco Caliendo** (DIW Berlin)**, Frank M. Fossen** (DIW Berlin)**, Alexander S. Kritikos** (EUV)**:** Risk Attitudes of Nascent Entrepreneurs – New Evidence from an Experimentally-Validated Survey. September 2006.

253. **Marco Caliendo** (DIW Berlin, IZA Bonn, IAB Nürnberg)**, Alexander S. Kritikos** (Europa-Universität Viadrina, GfA Berlin, IAB Nürnberg)**, Frank Wießner** (IAB Nürnberg): Existenzgründungsförderung in Deutschland - Zwischenergebnisse aus der Hartz-Evaluation. November 2006.

254. **Alfred Kötzle, Michael Grüning, Oleksandra Vedernykova:** Unternehmenspublizität aus Sicht der Praxis. November 2006.

255. **Friedel Bolle and Yves Breitmoser:** On the Allocative Efficiency of Ownership Unbundling. November 2006.

256. **Friedel Bolle and Yves Breitmoser:** Are Gas Release Auctions Effective?. November 2006.

257. **Karl Kurbel:** Process Models and Distribution of Work in Offshoring Application Software Development. Januar 2007.

258. **Friedel Bolle and Rostyslav Ruban:** Competition and Security of Supply: Let Russia Buy into the European Gas Market! Februar 2007.

259. **Marco Caliendo** (DIW Berlin) **and Alexander S. Kritikos** (Europa-Universität Viadrina): Is Entrepreneurial Success Predictable? An Ex-Ante Analysis of the Character-Based Approach. März 2007.

260. **Stephan Kudert und Ivonne Kaiser** (Europäisches Wissenschaftszentrum am Collegium Polonicum): "Die Unternehmenssteuerreform 2008: Eine Untersuchung zur Existenz von steuerlichen Lock-in-Effekten“. Mai 2007.

261. **Knut Richter and Irena Okhrin:** Solving a production and inventory model with a minimum lot size constrain. September 2007.