On a Class of Threshold Public Goods Games
With Applications to Voting and the Kyoto Protocol

Friedel Bolle

European University Viadrina Frankfurt (Oder)
Department of Business Administration and Economics
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Abstract: The launch of a public project requires support from "enough" members of a group. Members (players) are differently important for the project and have different cost/benefit relations. There are players who profit and players who suffer from the launch of the project. Examples are the Kyoto protocol, voting with different weights (shareholders, the UN with the veto power of the Security Council members), and international scientific or military expeditions. As coordination on one of the usually many pure strategy equilibria is difficult, mixed strategy equilibria are the focus of this investigation. If all players profit from the launch of the project then, despite the "unnecessary" costs, the requirement of full contributions is a Pareto-improvement to every original threshold. The contribution probabilities of some player types defined by their importance are characterized according to their cost/benefit relations.

Keywords: Threshold Public Goods, Provision Point Mechanism, Voting

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Europa-Universität Viadrina Frankfurt (Oder)

Postfach 1786

D - 15207 Frankfurt (Oder), Germany

Email: bolle@euv-frankfurt-o.de

1. Introduction

In a Threshold Public Good game the players contribute to a public project which is launched if and only if a certain level of contributions is reached. Contrary to the normal linear Public Good game with dominant strategies of zero contributions, there are many equilibria with certain or probabilistic contributions. In a special class of Threshold games players have only the binary choice of contributing their complete endowment or not, i.e. they join an enterprise or not, they cast a positive vote or not, they help establish or prevent a new golf resort by selling their site or not. Often Provision Point Mechanisms or Assurance Contracts allow only binary choices and thus belong to this class of games.

Models of voting or of Threshold public goods which have so far been investigated are mostly characterized by symmetry of weights of the players as well as symmetry of cost/benefit relations. In this paper I try to establish a framework for the analysis of Threshold games with a general definition of thresholds and asymmetric cost/benefit relations. Players are characterized according to their relative or absolute importance for meeting the threshold. Relations between contribution probabilities, importance of players, and cost/benefit relations are reported. As this is a first approach (and also under the impression of the large number of equilibria) relations cannot be completely characterized, however. In addition, a simple but important welfare implication is reported, namely the superiority of full contribution requirements even though they imply unnecessary costs if less than full contributions are needed for the launch of a project.

The most simple and prominent game in the class of binary Threshold games is the Volunteer's Dilemma (Diekmann, 1985, 1993) where only one player has to contribute his resources. Otsubo and Rapoport (2008) investigate a Volunteer's Dilemma game with T stages, where all players can decide in every stage whether to volunteer or not. The game ends if one player volunteers or when T is reached without a volunteer. Bilodeau and Slivinski (1996) and Weesie (1993, 1994) investigate this problem with continuous time. The sequential game with an exogenous order of players has been investigated by Bolle (2011). In addition to these theoretical investigations there are several experimental studies investigating the Volunteers Dilemma (Diekmann, 1986; Franzen, 1995; Goeree et al., 2005a;

Bolle, 2011). The other extreme Threshold game is the generalized Stag Hunt game where all players have to contribute to reach the threshold.

For general Threshold games there are mainly experimental studies, as far as they are not equivalent to voting models. Binary contributions are investigated by Dawes et al. (1986), Palfrey and Rosenthal (1991), Erev and Rapoport (1990),Chen et al. (1996), Croson and Marks (2000), Rose et al. (2002), Goren et al. (2003), Goeree and Holt (2005b), and McEvoy (2009). There are a lot more experiments with non-binary decisions. General results for Threshold games are that sequential contributions are more effective than simultaneous contributions and that refunds and rebates of insufficient and superfluous contributions improve the contribution probability.

Bagnoli and Lipman (1989) show that Threshold games have an efficient result if players are refunded and if only pure strategies are regarded. Palfrey and Rosenthal (1984) investigate a Threshold game with identical players which is very close to a voting model. Offerman et al. (1998) and Goeree and Holt (2005) substitute mixed strategy equilibria by Quantal Response equilibria which describe Nash equilibria when the precision parameter becomes infinitely large. Otherwise there are mainly voting models offering theoretical results.

The classic game theoretic voter model has been introduced by Downs (1957) and has been formalized and supplemented by Riker and Ordeshook (1968). They assume binary decisions (voting or not) and a voter's utility to be $U = qB - C + D$ with q = probability that his vote is decisive, B= utility difference because of the decisive vote, C= costs of voting. D = citizen's duty or psychological benefit from voting has been added and emphazised by Riker and Ordeshook (1968), mainly to explain the Paradox of Voting, i.e. a large percentage of people voting in a general election although q is tiny. For other attempts to explain the Voter Paradox see the survey by Geys (2006).

Two types of voting models may be distinguished, one where relative votes count and one where an absolute quota (e.g. minimal number of signatures for a referendum) has to be met. These are different, however, only if decisions are not binary[1]. Palfrey and Rosenthal (1983) show that, with binary decisions, the Voters' Paradox need not apply but that also for large electorates there are equilibria with mixed as well as pure strategies where individual votes keep a high level of decisiveness. One crucial difficulty is the coordination of players playing mixed and pure strategies.

Threshold public good games usually have a large number of pure strategy equilibria. The selection of one of these (i.e. coordination for the players) is difficult, however, because neither the Pareto-criterion nor other criteria apply. The situation with mixed strategy equilibria (if they exist) is often easier as there are usually less of them, as they often have nice properties as symmetry if players are symmetric, and as they may be easier ranked. On the other hand, equilibria where all players use strict mixed strategies need not exist and if they exist and are plausibly selected we know from many examples (e.g. Tsebelis,1990; Diekmann, 1993) that they seem to be plagued by "implausible" relations of mixture probabilities when comparing asymmetric players or evaluating the effect of parameter changes. Before we start with the theory section I want to show that such "implausible" results are almost inevitable.

	Launch	No launch	Support costs	Non-support costs
Utilities/costs	G_i	0	c_i	0

Table 1: Utilities (costs) in a Threshold Public Goods Game with players who profit from the launch of the project, $G_i > c_i > 0$, and players who do not, $G_i < c_i < 0$.

Regard the situation in Table 1 where utilities from a certain project are reported. This project has to be supported by a minimal set of players before it is launched. There may be other players with relations of costs and benefits different from those in Table 1, but those players have dominant pure strategies (see next section). Let us assume that all these players have already been eliminated from the game and the necessary sets of supporters have been adjusted. We are left with players who profit from the launch of the project and have costs of supporting < profit (the group which is advantaged after the launch of the project) and with players who suffer from the

[1] In most voter participation models, the binary decision is to vote or not to vote. In the latter case, the vote is endogenized. Then voting by supporters as well as non-voting by opponents can be counted as supportive and vice versa.

launch of the project but have negative costs of supporting > (negative) profit. All players would like to free-ride on the contributions or non-contributions of others.

It seems to be "plausible" that the players of the advantaged group will support the project with large probability if their cost/benefit relation c_i/G_i is low while players from the disadvantaged group should "plausibly" use low support probabilities if c_i/G_i is low. In mixed strategy equilibria, however, support probabilities depend only on the cost/benefit relations and not on the sign of costs and benefits. Therefore we inevitably have a "plausible" and an "implausible" dependency, independent of the result we find[2]. Therefore, mostly, we should not discuss results under the aspect of plausibility – unless we use this Janus-headed regularity as a reason to reject mixed strategy equilibria completely. Of a different quality are results which are plausible or implausible for both groups.

In the next section the theory is presented. Sections 3 and 4 report two applications and Section 5 is the conclusion. All proofs of Propositions as well as definitions and preparatory lemmas connected with these proofs are relegated to the appendix.

2. Theory

2.1 General Theory and why it is better to require more

In the **Threshold game**, there are $n \geq 2$ players $N = \{1, \dots, \mathrm{n}\}$ who can simultaneously contribute or not a predetermined (not necessarily identical) amount of a resource to a public project. If there are "enough" contributions then the project is launched. Note that the set of players and the determination of their required contributions have happened in a pre-game phase. Also the preferences of the players may have been influenced during this time. We will comment on the pre-game phase, but our analysis is restricted to the Threshold game.

Definition 1: $\mathcal{H}$ designates the set of subsets of N whose contributions suffice to produce the public good. It has the following properties: $\phi \notin \mathcal{H}$, $N \in \mathcal{H}$. If $S \subset S' \subset N$ and $S \in \mathcal{H}$ then also $S' \in \mathcal{H}$. We call $S \in \mathcal{H}$ a **minimal supporting set** if no strict subset of S is contained in $\mathcal{H}$. We call $S \subset N$ with $N - S \notin \mathcal{H}$ a **minimal blocking**

[2] There is no contradiction from the fact that we can describe every situation in a positive or in a negative frame. Reformulation in another frame leaves the incentives unchanged while the two groups in Table 1 have opposite incentives.

set if for no strict subset $S' \subset S$, $N - S' \notin \mathcal{H}$. If $\mathcal{H} = \{N\}$ we say that **full contributions are required**.

An example of $\mathcal{H}$, in particular if players have identical resources, are all subsets of N with k players.

Assumption 1: Player i bears costs c_i if he contributes to the project and he enjoys benefits G_i if the project is launched. There is zero utility if the project is not launched and zero costs if a player does not contribute, i.e. as in Table 1. Benefits may be influenced also by altruistic or envious feelings towards the other players. **Risk Neutrality** and **Complete Information** about $\mathcal{H}$ and about costs and benefits are assumed.

In cases of “real” public goods $G_i > c_i > 0$ applies. We will, however, take into account also other cases. If $G_i > 0 > c_i$, it is a dominant strategy to invest; if $c_i > G_i > 0$ then it is a dominant strategy not to invest. But there may also be losers with $G_i < 0$ if the project is realized. For these players $0 > c_i > G_i$ describes the case without a dominant strategy. If $0 > G_i > c_i$, it is a dominant strategy to invest. If $c_i > 0 > G_i$, it is a dominant strategy not to invest. But if $0 > c_i > G_i$, then i wants to free-ride on the decisions of others not to invest. So there remain only two interesting cases and we can simplify the game by disregarding all players with dominant strategies but taking the consequences of their behavior into account in $\mathcal{H}$.

Assumption 2: $N = N^+ + N^-$ where for all $i \in N^+$ we have $G_i > c_i > 0$ and for all $i \in N^-$ we have $0 > c_i > G_i$. The number of players in the two sets are n^+ and n^-.

In cases $n^- = 0$ there are as many pure strategy equilibria with the launch of the project as there are minimal supporting or sets. In such an equilibrium all $i \in S \in \mathcal{H}$, with $S =$ minimal set, contribute and all other players do not. Usually $\mathcal{H}$ contains more than one minimal set. In the case of identical contributions, there are $\binom{n}{k}$ minimal sets and pure strategy equilibria. If $\{i\} \notin \mathcal{H}$ for all i, then zero contributions by all i is another equilibrium which is Pareto-dominated by the asymmetric pure strategy equilibria. Respective equilibria exist for $n^+ = 0$. Cases with $n^- > 0$ and also $n^+ > 0$ are different because pure strategy equilibria need not exist. In a pure strategy equilibrium, the project is launched with probability 1 or 0. If launched with probability 1 then all $i \in N^-$ would also contribute because of $c_i < 0$. This possibly allows some $i \in N^+$ to withdraw their support until the set of supporters is minimal. But then there is an incentive for every decisive player $i \in N^-$ not to support the project any longer.

Proposition 1: (i) In a pure strategy equilibrium with the launch of the project, support is provided by players from a set S with $N^- \subset S,\ \ S \in \mathcal{H}$, $S - \{i\} \notin \mathcal{H}$ for $i \in S - N^-$, and $S - \{i\} \in \mathcal{H}$ for $i \in N^-$. (ii) In a pure strategy equilibrium without the launch of the project support is provided by players from S' with $S' \subset N^-$, $' \notin \mathcal{H}$, $\{i\} \cup S' \in \mathcal{H}$ for all $i \in N^- - S'$, and $\{i\} \cup S' \notin \mathcal{H}$ for $i \in N^+$.

We now ask which properties equilibria in mixed strategies (or in mixed and pure strategies) have. We assume that players i=1,…,n contribute with probabilities p_i. Some of these probabilities may be 0 or 1. Sometimes we concentrate on the set of players with mixed strategies, i.e. $\mathcal{H}$ describes the necessary contributions if all pure strategy players (not only those with dominant strategies) have already been taken into account.

Definition 2: Let S be the (stochastic) set of contributing players and let us define Q_{-i} as the probability that the project is launched also without player i's contribution, i.e. if other players contribute with probability p_j, then

$$Q_{-i} = prob(\{S: i \notin S,\ S \in \mathcal{H}\}) = \sum_{S \subset N-\{i\}, S \in \mathcal{H}} \prod_{j \in S} p_j \prod_{j \in N-S-\{i\}} (1-p_j)$$

Let us define Q_{+i} as the probability that the project is launched if i contributes, i.e.

$$Q_{+i} = prob(\{S: i \notin S,\ \{i\} \cup S \in \mathcal{H}\}) = \sum_{S \subset N-\{i\}, S \cup \{i\} \in \mathcal{H}} \prod_{j \in S} p_j \prod_{j \in N-S-\{i\}} (1-p_j).$$

Then $q_i = Q_{+i} - Q_{-i}$ is the probability that i's contribution is decisive. □

Player i's expected utility is

$$(1)\quad U_i = G_i * [(1-p_i) * Q_{-i} + p_i * Q_{+i}] - p_i d_i$$

$$= G_i * Q_{-i} + p_i * [G_i * q_i - d_i]$$

with $d_i = c_i$ if there are no refunds (if contributions are insufficient) and rebates (if contributions are superfluous), and $d_i \le c_i$ if there are. d_i is independent of p_i.

$$(2)\; d_i(N) = d_i = \begin{cases} c_i & \text{without refunds and rebates} \\ c_i * Q_{+i} & \text{with refunds but not rebates} \\ c_i * \sum_{S\cup\{i\}\in\mathcal{H}} prob(S)B(S) & \text{with refunds as well as rebates} \end{cases}$$

with $B(S) = 1$ if S is a minimal supporting set and B(S) ≤1 otherwise. In “refunds but not rebates” we have $B(S) = 1$ for all $S \cup \{i\} \in \mathcal{H}$. Note that $i \in N^+$ are happy about refunds and rebates, while, because of $c_i < 0$, $i \in N^-$ do not like them. From their point of view the equivalence to refunds and rebates are payments to those who have not contributed to the project.

A mixed strategy equilibrium with $0 < p_i < 1$ requires that U_i is independent of p_i, i.e.

$$(3)\; \partial U_i / \partial p_i = G_i * q_i - d_i \equiv 0.$$

This requirement is standard in the literature on voting and threshold public goods, probably the first time derived by Downs (1957). The following simple welfare implications, however, seem to have been unnoticed. Inserting (3) into (1) provides us with the equilibrium utility which i expects if he plays a mixed strategy.

$$(4)\; U_i = G_i * Q_{-i}$$

$$= G_i * Q_{+i} - d_i.$$

If, in equilibrium, $G_i * q_i - d_i > 0$ then $p_i = 1$ and (1) implies $U_i < G_i * Q_{+i} - d_i$; if $G_i * q_i - d_i < 0$ then $p_i = 0$ and $U_i > G_i * Q_{+i} - d_i$.

Proposition 2:

(i) $p_i = 0\ (p_i = 1)\ if\ q_i < (>) d_i/G_i$.

(ii) Higher refunds decrease the equilibrium expectation q_i that one’s own contribution is critical. For $\{i\} \in \mathcal{H}$ this implies higher Q_{-i}, i.e. i expects others “on average” to increase their contribution probabilities.

If $n^- = 0$ then

(iii) If full contributions are required there are three equilibria: E_0 with $p_i = 0$ and $U_i = 0$ for all I, E_1 with $p_i = 1$ and $U_i = G_i - d_i$ for all i, and a mixed strategy equilibrium E_m with $U_i = 0$.

(iv) If (3) applies for i then the expected equilibrium utilities of i are not larger than the equilibrium utilities in E_1. If, in addition, $\{i\} \in \mathcal{H}$ then the expected equilibrium utilities of i are equal to the equilibrium utilities in E_1.

(iii) and (iv) can, of course, be equivalently formulated for $n^{+} = 0$. From (i) follows that $p_i = 0$ if $q_i = 0$ and $p_i = 1$ if $q_i = 1$. Proposition 2 (iv) does not prove the existence of mixed strategy equilibria but describes properties of equilibria if they exist. (For symmetric cases with complete information which is the standard case in the literature we will derive equilibria below and find that a symmetric mixed strategy equilibrium does not always exist.) (iv) means that, except in the Volunteer's Dilemma (k=1 and the generalization $\{i\} \in \mathcal{H}$), the requirement that all contribute even if full contribution is not necessary for the production of the public good, is a Pareto-improvement for all those with positive equilibrium probabilities. If there are players who contribute with probability zero then these are possibly worse off after a switch to the requirement of full contribution; all other players are better off. Note that the inferiority of intermediate thresholds is not self-understanding. Full contributions require unnecessarily high costs, but the losses according to attempted free riding are larger. In a symmetric game, (4) implies that at least in the case k=1 contribution probabilities increase with the amount of refunding and, more generally, with decreasing cost/benefit relations.

While at least in cases $n^{-} = 0$ and $n^{+} = 0$, any minimal supporting set or any minimal blocking set can be the basis of pure strategy equilibria and while also Proposition 1 characterizes pure strategy equilibria independent of costs and profits, in mixed strategy equilibria these incentives are central. When the launch of the project is uncertain then many players may be critical with a certain probability, i.e. $0 < q_i < 1$. Also players with zero or one contribution probabilities face restrictions $q_i < d_i/G_i$ or $q_i > d_i/G_i$ while in pure strategy equilibria the respective restrictions (with $q_i = 0$ for outside players and $q_i = 1$ for inside players) are always fulfilled. These restrictions prevent certain constellations of mixed/pure strategy equilibria but they need not prevent all of them, not even in symmetric situations. If there is a set of mixing players and a set of players who do not contribute, then this situation can be "stabilized" if an outside player faces a lower probability q_i than a inside player.

2.2 Player relations and contribution probabilities

Definition 3: Player i is said to be **not less important** than j if, for every $S \subset N - \{i,j\}$ and $S \cup \{j\} \in \mathcal{H}$ also $S \cup \{i\} \in \mathcal{H}$ applies. i and j are said to be in **symmetric strategic positions** if i is not less important than j and j is not less important than i.

Note that two players can have opposite incentives, namely if one is member of N^+ and the other of N^-, but nonetheless be in symmetric strategic positions and have the same cost/benefit relation $r_i = \frac{d_i}{G_i} = \frac{d_j}{G_j} = r_j$.

Lemma 1: If all pairs of players are in symmetric strategic positions then the threshold can be formulated as "k of n have to contribute".

Let us now turn to the following questions: If i is not less important than j, will he contribute with larger or with smaller probability than j? Do players with larger cost/benefit relations contribute with larger or with smaller probability?

In pure strategy equilibria both questions are largely irrelevant. If $n^- = 0$ then every minimal supporting set is the basis of a pure strategy equilibrium. Importance may play a role only insofar as it is related to the minimal supporting sets. These equilibria are completely independent of cost/benefit relations.

Let us now regard mixed strategy equilibria. We will see in the following that the two questions have different answers under different circumstances. Proposition 3 below will describe a "main case", but we find different relations in other cases.

Proposition 3: Assume that all players $h \in N$ play mixed strategies with $0 < p_h < 1$ and that $\{i,j\}$ is contained in a minimal supporting set.

(i) If players i and j are in symmetric strategic positions and if $r_i > (=,<) r_j$ then $p_i < (=,>) p_j$.

(ii) If i is not less important than j and if $r_i \le r_j$ then $p_i \ge p_j$.

Examples: $N = \{1,2\}$, $\mathcal{H}_1 = \{\{1,2\}\}$, $\mathcal{H}_2 = \{\{1\},\{2\},\{1,2\}\}$. In these two examples the two players are in symmetric strategic positions. In the case of $\mathcal{H}_1$ we can apply Proposition 3 and get higher contribution probabilities for the player with the lower cost/benefit relation. (See also the proof of Proposition 2 (iii) with the derivation of the mixed strategy equilibrium in the full contribution case.) In the case of $\mathcal{H}_2$ we cannot apply Proposition 2 but we can compute the mixed strategy equilibrium directly and find $p_i = 1 - r_j$, i.e. we find higher contribution probabilities for the player with the higher cost/benefit relation. (See also the equilibrium probabilities derived in the asymmetric Volunteer's Dilemma; Diekmann, 1993). The latter are "paradoxical" (Diekmann, 1993) in the case of $N = N^+$; in the case of $N = N^-$, however, they are "plausible". The opposite characterization applies to $\mathcal{H}_1$. Note that both cases as well as $\{1\} \in N^+$and $\{2\} \in N^-$ have the same unique mixed strategy equilibrium.

Proposition 3 highlights that the relationship between cost/benefit relations and contribution probabilities is, in "most" cases, contrary to the relationship in the Volunteer's Dilemma.

Let us now investigate two extreme cases, one where a certain player is extremely important and one where a player may be extremely unimportant. In the latter case, we will deal with another class of games where Proposition 3 is not applicable.

Definition 4: A player i is **irreplaceable** if $i \in S$ for every $S \in \mathcal{H}$.

Military intervention by NATO in a larger country is not possible without the United States who are then an irreplaceable player. A cartel often cannot be formed without the two largest firms who are then both irreplaceable players. The Security Council of the UN consists of irreplaceable players. In all cases the coalitions $S \in \mathcal{H}$ can be larger.

Proposition 4:

(i) An irreplaceable player is not less important than any other player.

(ii) If an irreplaceable player i is refunded then contribution with $p_i = 1$ is an optimal strategy.

Assume that i and j are both mixed strategy players with equilibrium contribution probabilities $0 < p_i, p_j < 1$. Then

(iii) If i is irreplaceable then $p_i \leq \frac{r_j}{r_i}$.

(iv) If i and j are irreplaceable then $\frac{p_j}{p_i} = \frac{r_i}{r_j}$.

(v) If i is irreplaceable then $U_i = 0$.

From (v) follows that, in a mixed strategy equilibrium, an irreplaceable player does not profit from his strong position; on the contrary, he is not better off than after playing the pure strategy of non-contribution. If $\mathcal{H} = \{N\}$, then all players are irreplaceable and (iv) and (v) apply for all players. We have discussed this situation already in Proposition 2 (iii) and selected the symmetric pure strategy equilibrium with contributions by all players.

If the coalition of all irreplaceable players is an element of $\mathcal{H}$, then it is the only element of $\mathcal{H}$, and when $\mathcal{H}$ has only one element then there are no difficulties with coordinating on the pure strategy equilibrium where only these players contribute.

Definition 5: A player **j is unnecessary in the face of i** if $\{i\} \in S \in \mathcal{H}$ implies $S - \{j\} \in \mathcal{H}$.

In many teams a specialist in one area is necessary for success, say one book-keeper in a non-profit organization which relies on voluntary work, but not two. One of them is unnecessary in the face of the other. If the coalition of all irreplaceable players is an element (necessarily the only) of $\mathcal{H}$ then all other players are unnecessary in the face of any irreplaceable player.

Proposition 5: Assume that j is unnecessary in the face of i. Then

(i) i is not less important than j,

(ii) $\{i, j\}$ are not contained in a minimal supporting set,

(iii) If $r_i < r_j$ then $p_j = 0$.

Generally we can assume players to have certain resources which they can supply to the project. The threshold can be described by a (production) function of these resources which has to surpass a certain limit. A simple case which describes many applications is an additive function.

Definition 3: If every player can be characterized by a number x_i and the threshold is defined by the requirement that the sum of the contributing players' x_i reaches a certain limit L then we call the **threshold additive**.

With an additive threshold a player i is not less important than j if $x_i \geq x_j$, he is irreplaceable if and only if $\sum_{j \neq i} x_j < L$.

2.3 A remark on dynamics

Simultaneous contributions take place in secret voting and in other examples. But there are also examples where the assumption of simultaneity (almost standard in voting models) is less appropriate. Deviations from this procedure are “distributed contributions”, where all players can contribute during a certain time interval or in the course of a certain number of periods, or “sequential contributions” where, in an exogenous or an endogenous order, one player after another can contribute. In both cases all players are immediately informed about new contributions. For the Volunteer’s Dilemma distributed contributions have been theoretically investigated by Otsubo and Rapoport (2008) for a finite number of periods and by Weesie (1994) for continuous time. Sequential contributions in the Volunteer’s Dilemma are investigated by Bolle (2011). In the rest of this sub-section we want to concentrate on distributed contributions for which open voting and the Kyoto protocol are prominent examples.

Let us assume that there are 2 periods in which every player can contribute. Contrary to Otsubo and Rapoport (1996) and Weesie (1994) we do not introduce exogenous incentives for early contributions but concentrate on the strategic effect of early contributions.

Let us assume that the situation in period 2 is as described in Definition 1 and Assumption 1 above and that a certain mixed strategy equilibrium has been selected. If i expects no other player to contribute in period 1, he faces the following alternatives:

(a) He does not contribute either. Then, in period 2, the mixed strategy is played and his expected utility is $U_i^- = G_i * Q_{+i} - d_i$. .

(b) He contributes. Then his utility is $U_i^+ = G_i * Q(N - \{i\}) - d_i$ with $Q(N - \{i\})$ describing the probability that, in an selected equilibrium with players $N - \{i\}$, enough players $S \subset N - \{i\}$ contribute to meet the threshold $S \cup \{i\} \in \mathcal{H}$.

In the case $\{i\} \in \mathcal{H}$ we find $Q_{+i} = Q(N - \{i\}) = 1$ and therefore $U_i^- = U_i^+$. In other cases the comparison of Q_{+i} and $Q(N - \{i\})$ is difficult. We come back to this problem in the case of symmetric equilibria. At least for irreplaceable players, early contributions are advantageous.

Proposition 6. Let Q_{+i} and $Q(N-\{i\})$ be defined as above and let us assume second round mixed strategy equilibria.

(i) If $Q_{+i} \geq Q(N-\{i\})$ for all i then an equilibrium in the game with distributed contributions exists where all players contribute only in the second round.

(ii) If $Q_{+i} < Q(N-\{i\})$ for at least one i then there is no equilibrium in the game with distributed contributions where all players contribute only in the second round.

In all cases which are described by Proposition 6 (ii) there are also asymmetric second round equilibria where just those players who have an incentive for early contribution play a pure strategy with contribution. Early contributions, however, facilitate the coordination with respect to this equilibrium.

3. Voting

Let us first make clear that voting is a special case of the Threshold game only under the assumption that the set of voters is known in advance and that abstention is not allowed, not effective (for example because an absolute and not a relative quota has to be fulfilled) or prohibitively expensive. Non-participation as a substitute of abstention is allowed if we endogenize the subsequent vote. Most models of voter participation apply this technique in order to cope with a binary decision situation. We may, otherwise, regard a two-step game where first the voter set is determined and then the resulting Threshold game (without abstention) is analyzed. With binary decisions, there is no difference between relative or absolute quotas. Note that, in voting, refunding is usually not possible.

Secret ballots as well as public votes may cause conflicts for some voters. Imagine that a parliament decides on a legislative proposal by a party, for example the abolition of the death penalty, entering a war, homosexual marriage, carbon taxes, etc. A member of the party which issued the proposal might be personally opposed to the issue but nonetheless he does not wish his party to be defeated. There are at least psychic costs $c_i > 0$ for him for supporting the issue by his vote. In a public vote he has to fear consequences in his electoral district where people want him to vote for or against the issue and he has to fear penalties from his party if he does not vote approvingly. Nonetheless approval may incur positive costs. A member of an opposing party (which wants the proposal to be rejected) may be in a similar situation. All members of a parliament may face a common conflict independent of their party affiliation when they vote on a legislative pay rise.

In voting, an absolute threshold is additive, (mostly) with $x_i = 1$ for all i and a necessary limit L=k. Exceptions are, for example, shareholder voting or the existence of a veto player. Note that k may take every number between 1 and n in (absolute) majority voting or voting with any other (absolute) quota because we have already eliminated all players with dominant pure strategies.

With all $x_i = 1$ all players are in equal strategic positions and therefore Proposition 3 (i) applies. If all cost/benefit relations are $\frac{d_i}{G_i} = r$ then all mixed strategies have the same $p_i = p$. We determine p by solving (3), i.e.

(5) $\partial U_i / \partial p_i = \binom{n-1}{k-1} \mathrm{p}^{\mathrm{k}-1}(1-\mathrm{p})^{\mathrm{n}-\mathrm{k}} - \mathrm{r} = 0.$

In the case k=1 the left hand side of (5) is monotonically decreasing in p from $1 - \mathrm{r} > 0$ to $-\mathrm{r} < 0$; in the case k=n it is monotonically increasing in p; for other k it is $-\mathrm{r}$ for $\mathrm{p} = 0$ as well as for $\mathrm{p} = 1$ and larger and single peaked between 0 and 1. The maximum value is taken at $\mathrm{p} = (\mathrm{k}-1)/(\mathrm{n}-1)$ and is not necessarily larger than 0. Therefore we have a unique solution of (5) if k=1 and k=n. If k=2,…,n-1, we have 2 or 1 (boarder case) or no solution.

In the case $n^- = 0$, k=2,…,n-1, and when there are two solutions, the lower p is connected with a Pareto-inferior equilibrium because lower p imply lower Q_{+i} and therefore lower utility for all players because of (4). If there is no solution of (5), there is no mixed strategy equilibrium with positive contribution probabilities. The only symmetric equilibrium consists of all players not contributing. At last, in the case k=n, we select the Pareto-superior and also symmetric pure strategy equilibrium where all players contribute. This equilibrium is discussed in Proposition 2 (iii).

In the case $n^+ = 0$, according to the same arguments, we should concentrate on the lower equilibrium and, in the case of no solution of (19), the equilibrium where all players contribute. In the case k=n, no player contributes.

If $n^+, n^- > 0$ then our criterion for the selection of equilibria according to Pareto-superiority does not help us. Players from N^+ favor the opposite of what players from N^- prefer. Proposition 1 shows that, in our example with identical cost/benefit relations, only one pure strategy equilibrium can exist, namely all members of N^- supporting the project and all members of N^+ not supporting. This is an equilibrium except for $n^- = k$ (where single members of N^- can prevent the launch of the project) and $n^- = k - 1$ (where a single member of N^+ can launch the project by deviating). These pure strategy equilibria are complete free-riding equilibria because those who profit from the project do not contribute and those who do not profit contribute. There are also equilibria where some players play pure strategies, for which (3) does not apply, and some play mixed strategies for which (3) applies. For these equilibria, again severe problems with coordination may exist.

k	1	2	3	4	5	6
p*	0.186	0.303	0	0	0.882	1
$Q(N)$	0.709	0.587	0	0	0.848	1
U_i	1.8	1.48	0	0	1.73	1.8
Q_{+i}	1	0.886	0	0	0.975	1
$Q(N - \{i\})$	1	0.724	0.639	0.643	0.852	1

Table 2: Equilibria with $G_i = 2.8$ and $c_i = 1$. Contribution probabilities p*, launch probabilities $Q(N)$, utilities U_i and Q_{+i} = prob(k-1 of n-1 contribute) for the problem with $n = n^+ = 6$, $k = 1, 2, \dots 6$. $Q(N - \{i\})$ for the problem with n-1 players and k-1.

Example: The example in Table 1 shows, first, what we already knew from Proposition 1, namely that the full contribution pure strategy equilibrium provides more utility than k=2,3,4,5 although more costs are incurred. In the cases k=3 and k=4 the symmetric equilibrium contribution probabilities are 0.

In an open vote with a show of hands to signal approval (e.g. in committee sessions) there is a certain time span for decisions and the early decisions of others can be observed. In the cases k=3 and k=4 every voter is better off if he contributes first and thus changes the game to one where k=2 or k=3 approvals from 5 voters are necessary. If we virtually divide the time span into two periods then, in the cases k=3, 4 the mixed strategy consists of a probability to contribute (show of hands) in the first period and, depending on the number of resulting votes, another probability of contributing in the second period. Because $Q_{+i} > Q(N - \{i\})$ Proposition 5 tells us that, for k=2 and k=5, no voter will contribute in the first period (quickly) but all in the second (delayed). In the cases k=1 and k= 6 the utility of the early contributing player remains the same. Note that such a symmetric mixed strategy equilibrium over several rounds can nonetheless end with k=3 or 4 in the last round where, then, all players contribute with 0 probability.

In particular in the case of secret votes with “medium” requirements of necessary approvals the proposing party should be alarmed. A way out of this dilemma may consist of making the majority narrower which is a disadvantage only at first glance. If convincing some of the mixed strategy players not to participate in the voting session is difficult because abstention is connected with particularly large costs the party may convince other members with a high party loyalty (who would vote approvingly with probability 1) not to participate. Thus, in our example, the required k may increase from 3 or 4 to 5 or 6 which implies a Pareto-improvement for all members of the proposing party. There are many statements by politicians and observers that narrow majorities foster the party discipline[3], but I must admit that I do not know of a case where majorities have purposefully been narrowed. Party whips work mainly by influencing G_i and c_i and aim at high participation rates of their delegates. We may doubt, however, that such a policy is always optimal.

If we alternatively assume $n^- = n^+ = 3$, then the probabilities in the cases k=1, 2, 5 are still equilibrium probabilities, in the cases 3, 4, they are not. In the cases k=2,5,6 the equilibrium selection according to Pareto-superiority does neither apply. Perhaps the pure strategy equilibrium described in Proposition 1 (all in N^- contribute, all in N^+ do not) is now the most plausible equilibrium. But again these equilibria do not apply for k=3 and k=4. Note, however, that the players from N^+ have the same incentive for early contributions as in the case $n^+ = 6$.

The situation k=3,4 in our example is not an exception. If the number of players increases then, except for k=1 and k=n, q_i necessarily approaches 0 and therefore (3) cannot be fulfilled. Palfrey and Rosenthal (1984) describe this as the

[3] “... the narrow majority can be a blessing since it offers its own incentives for Members to vote with the party” (Peters, 2004, p.233)

disappearance of mixed strategy equilibria for large groups. But as in the case of our example there may be incentives for early voting and there is a multitude of pure/mixed strategy equilibria. These equilibria are described by Palfrey and Rosenthal (1983).

4. The idea of the Kyoto Protocol

The Kyoto Protocol to the United Nations Framework Convention on Climate Change (UNFCCC) sets binding obligations on industrialized countries to reduce emissions of greenhouse gases. The Protocol was adopted by parties to the UNFCCC in 1997. Article 25 of the Protocol specifies that the Protocol enters into force "on the ninetieth day after the date on which not less than 55 Parties to the Convention, incorporating Parties … which accounted in total for at least 55% of the total carbon dioxide emissions for 1990 …, have deposited their instruments of ratification, acceptance, approval or accession." (The United Nations, 1998) The 55 nations limit was easily passed but the required 55% of the carbon dioxide emissions of 1990 turned out be difficult to reach as some large polluters, in particular the USA, did not ratify the protocol. Only after lengthy negotiations and heavy discounts, Russia joined the club so that, ultimately, the treaty entered into force in 2005.

The idea of the Kyoto protocol is to change a normal public goods game with costs c_i and benefits $g_i < c_i$, whose unique equilibrium is non-contribution, into a Threshold game with $G_i = \sum g_i > c_i$ and contribution equilibria. By introducing a medium threshold of 55%, however, much or all of the improvement has been lost again. Probably the “fathers” of the Kyoto protocol wanted to launch worldwide climate policy successfully and probably they thought it would be easier to reach a medium threshold than a high one - but we know from Proposition 2 that the contrary is true. A second disadvantage of a medium threshold is that the probability q_i that i’s contribution is decisive is, at least for small players, rather low which makes non-contribution a dominant strategy for these countries.

How should a workable contract such as the Kyoto protocol look? First, negotiations according to the reduction obligation of a country should take into account that the contract will not be ratified if a majority in country i believes $G_i < c_i$. Only a small number of countries can be assumed to be altruistic and take also other countries’ advantages into account; to plead for such a point of view may be successful on a climate conference but mostly not in a parliamentary decision in the home country. Second, the contract should be signed only by those who (almost) certainly fulfill $G_i > c_i$. Third, with a highly demanding threshold, big players as China, USA, and EU become irreplaceable and, with refunding, will contribute with certainty. This requires that the first two points are met and that refunding is not deteriorated by a too long period in which countries have time to ratify the contract. In such a contract, refunding means “no obligation if the necessary quota is not reached”. If investments in CO2 have taken place, however, they are practically irreversible. Theorem 4(v) shows that, without refunding, irreplaceable players are in a difficult position.

5. Conclusion

A Threshold Public Good game has been investigated with binary decisions, a general Threshold defined on contributing player sets, players who profit and players who suffer from the launch of the project and have different cost/benefit relations. If all players profit from the launch of the project then it is better to establish an “unnecessarily” high threshold because the additional costs are overcompensated by the increasing probability of success. Players in equal strategic positions with equal cost/benefit relations are supportive with equal probabilities. If player i is not less important than j and has a smaller cost/benefit relation then he supports the project with a higher probability, provided the two players are members of a minimal supporting set. In the Volunteer’s Dilemma, however, where the minimal supporting sets are singletons players with a lower cost/benefit relation support the project with a lower probability. If player j is even unnecessary if i supports the project, then j’s support probability is zero. An irreplaceable player (who is contained in every minimal supporting set) is, in a mixed strategy equilibrium, not better off than with playing the pure strategy of zero contributions. With refunds, an irreplaceable player contributes with certainty.

These are important messages for real world Threshold games although we know from experimental economics that people often do not behave according to theoretical results. But I think that the message for climate policy, if it wants to rely on Kyoto protocol-like treaties, is nonetheless important: Establish not so ambitious requirements from single countries in order to maintain low cost/benefit relations but require rather ambitious thresholds (close to 100%) for the contract to enter into force.

A certain shortcoming of the analysis is its reliance on Complete Information. But let us ask where, in the case of the Kyoto-protocol for example, the deviations are. There are a lot of “objective” scientific institutes and organizations which estimate costs and benefits for all countries and most of them can be trusted not to manipulate their estimates in favor of some countries. Of course, all these figures are highly uncertain and, insofar, there is incomplete information. But there is, perhaps with the exception of some countries like China, no substantial amount of private information. So we have mainly a collective decision under risk but not one where private information has strategic value.

There are other examples of the provision of Threshold Public Goods where private information can be decisive, for example tight parliamentary votes where secret incentives depending on threats, bribes, and tit-for-tat arrangements may be decisive. It does not look easy, however, to merge mixed strategy equilibria and Incomplete Information.

Appendix

A1. Proposition 1:

Without proof.

A2. Proposition 2:

Proof: (i) follows from (3). (ii) Higher refunds decrease d_i and therefore, because of (3), the expected equilibrium q_i. Because $Q_{+i} = 1$ for $\{i\} \in \mathcal{H}$ the decrease of q_i must be due to an increase of Q_{-i}. (iii) $p_i = 1$ is the best reply to full contributions of others because $G_i > c_i$, and $p_i = 0$ is a best reply to others' non-contributions (in the cases with WR the best replies are arbitrary). In every mixed strategy equilibrium we have $q_i = P/p_i = d_i/G_i$ with $P = \prod_{j \in N} p_j = \left[\prod_{j \in N} d_i/G_i\right]^{1/(n-1)}$, i.e. the mixed strategy equilibrium is unique. Because $Q_{+i} = q_i$ (4) implies $U_i = 0$. (iv) follows from $Q_{+i} = 1$ if $\{i\} \in \mathcal{H}$ and $Q_{+i} \leq 1$ otherwise. □

A3. Lemma 1:

Proof: If $S \in \mathcal{H}$ then every player can be substituted by a player from $N - S$. I.e. if a set with k players is in $\mathcal{H}$ then every set with k players is in $\mathcal{H}$. □

A4. Preparatory definitions and Lemmas

Let us now assume two players with contribution probabilities p_i and p_j and let us define

(6) $q_{i,j} = prob(\{S: i,j \notin S \notin \mathcal{H},\ \{i,j\} \cup S \in \mathcal{H}\})$

which is the probability that the coalition $\{i,j\}$, regarded as one player, is decisive.

(7) $q_{i,-j} = prob(\{S: i,j \notin S \notin \mathcal{H}, \{i\} \cup S \in \mathcal{H}\})$

describes the probability that i is decisive when j does not contribute and, vice versa,

(8) $q_{j,-i} = prob(\{S: i,j \notin S \notin \mathcal{H}, \{j\} \cup S \in \mathcal{H}\})$.

In addition, we define

(9) $q_{i,+j} = prob(\{S: i,j \notin S, \{j\} \cup S \notin \mathcal{H}, \{i,j\} \cup S \in \mathcal{H}\}) = q_{i,j} - q_{j,-i}$

which describes the probability that i is decisive when also j contributes. Respectively we get

(10) $q_{j,+i} = prob(\{S: i,j \notin S, \{i\} \cup S \notin \mathcal{H}, \{i,j\} \cup S \in \mathcal{H}\}) = q_{i,j} - q_{i,-j}$.

After these preparatory definitions we get

(11) $q_i = (1 - p_j) * q_{i,-j} + p_j * q_{i,+j}$

(12) $q_j = (1 - p_i) * q_{j,-i} + p_i * q_{j,+i}$.

Apparently the following relations hold

(13) $q_{i,-j} + q_{j,-i} \leq q_{i,j}$,

(14) $q_{i,-j} = q_{j,-i} = q_-$ and $q_{j,+i} = q_{i,+j} = q_+$ if i and j are in symmetric strategic positions

(15) $q_{i,-j} \geq q_{j,-i}$ if i is not less important than j.

Lemma 2: Assume that i and j are both mixed strategy players with equilibrium contribution probabilities $0 < p_i, p_j < 1$. If $r_i > (=,>) r_j$ then

(16) $q_{i,-j} - q_{j,-i} < (=,>)(q_{i,j} - q_{i,-j} - q_{j,-i}) * (p_i - p_j)$.

Proof: After applying (9) and (11) and then (7) and (8) to $q_i = r_i < (=,>) r_j = q_j$ we get (16).

Because of (15) and (13), Lemma 2 allows us to investigate the effect of cost/benefit relations on price relations. It is decisive, however, whether (13) applies with equality or not.

Lemma 3: If all players $h \in N$ play mixed strategies with $0 < p_h < 1$ and if $\{i, j\}$ is contained in a minimal supporting set then $q_{i,-j} + q_{j,-i} < q_{i,j}$. If $\{i, j\}$ is not contained in any minimal supporting set then $q_{i,-j} + q_{j,-i} = q_{i,j}$.

Proof: If all players play strict mixed strategies then all sets $S \subset N$ of supporting players are assumed with positive probability. Let us define $q_{+i,+j} = prob(\{S: i, j \notin S, \{j\} \cup S \notin \mathcal{H}, \{i\} \cup S \notin \mathcal{H}, \{i, j\} \cup S \in \mathcal{H}\})$ which is positive if $\{i, j\}$ is contained in a minimal supporting set and zero otherwise. As $q_{i,j} = q_{i,-j} + q_{j,-i} + q_{+i,+j}$, (12) has to apply without equality in the former case and with equality in the latter.

A5. Proposition 3

Proof: (i) If players i and j are in symmetric strategic positions then $q_{i,-j} = q_{j,-i}$. (16) and Lemma 3 imply the statements. (ii) If i is not less important than j then $q_{i,-j} \geq q_{j,-i}$ and (16) and Lemma 3 imply $p_i \geq p_j$.

A6. Proposition 4

Proof: (i) Compare definitions. (ii) follows from $U_i = p_i * Q_{+i} (G_i - c_i)$ in the case of refunding of an irrepleaceable player, i.e. with $Q_{-i} = 0$. (iii) If i is irreplaceable then $q_{j,-i} = 0$ for all j. (11) and (12) imply

(17) $r_j = p_i * q_{j,+i} \geq p_i * q_{i,j}, \quad r_i = (1 - p_j) * q_{i,-j} + p_j * q_{i,+j} \leq q_{i,j}$.

By dividing the two inequalities we get $p_i \leq \frac{r_i}{r_j}$. (iv) If also j is irreplaceable then also $q_{i,-j} = 0$ and instead of (17) we get

(18) . $r_j = p_i * q_{i,j}, \ r_i = p_j * q_{i,j}$.

(v) follows from $Q_{-i} = 0$ and (4).

A7. Proposition 5

Proof: (i) and (ii): Compare definitions. (iii) From (ii) and Lemma 3 follows $q_{i,j} = q_{i,-j} + q_{j,-i}$. Let us assume $0 < p_i, p_j < 1$. Then we would get $r_i = q_{i,-j}$ and $r_j = q_{j,-i}$. Because i is not less important than j, $q_{j,-i} \leq q_{i,-j}$. This is a contradiction to $r_i < r_j$. I.e. either I or j plays a pure strategy. Now assume $p_i < p_j$. Then $p_j > 0$ and therefore $q_j = q_{j,-i} \geq r_j$, and $p_i < 1$ and therefore $q_i = q_{i,-j} \leq r_i$. Because of $q_{j,-i} \leq q_{i,-j}$ we get $r_i \geq r_j$ which is again a contradiction to the assumption $r_i < r_j$. Therefore $p_i \geq p_j$ and, because not both of them can play mixed strategies, $(p_i, p_j) = (1, p) \; or \; (p, 0)$ with $0 \leq p \leq 1$ have to apply. But if i contributes with certainty then j's contribution is superfluous and he will contribute with zero probability.

A8. Proposition 6

Without Proof.

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